Method for Minimizing Total Generalized Squared Correlation of Synchronous DS-CDMA Signature Sequence Sets in Multipath Channels

Paul Cotae and Matt Aguirre

Abstract - **We characterize the Total Generalized Squared Correlation (TGSC) for a given signature sequence set used in uplink synchronous code division multiple access (S-CDMA) when channel state information is known perfectly at both transmitter and receiver. We give a definition of the TGSC based on the eigenvalues of Gram matrix associated to signature sequences set for multipath channels in the presence of the colored noise. Total Squared Correlation (TSC) and Total Weighted Squared Correlation (TWSC) measures are particular cases of TGSC. We present a method for minimizing TGSC (TSC, TWSC) in multipath channels and in the presence of the colored noise. Numerical results for overloaded synchronous CDMA systems are presented in order to support our analysis.**

***Index terms*: total squared correlation (TSC), total weighted square correlation (TWSC), Welch Bound Equality (WBE) sequences, CDMA codeword optimization and Gram matrix.**

I. Introduction

In code-division multiple access (CDMA) systems with perfect power control, the major limitation in performance is due to multiple access interference (MAI). This interference is the result of the correlation among the users' signature sequences and it can be minimized (or eliminated) by designing signature sequence sets with low (or zero) correlation values. Much research has been conducted in the area of multiuser detection to suppress/cancel/avoid interference for a deterministic signature sequence sets [5], [14], [17], [26], [27], [28] and references therein.

Recently, there has been interest in managing interferences in a CDMA system from the transmitter side. When channel state information (CSI) is available at the transmitter an effective transmit power allocation may improve error rate performance or increasing the theoretical information capacity by decreasing the interferences in the system.

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Prof. Paul Cotae is with the University of Texas at San Antonio, TX, 78249, USA, Ph# 210-458-6269; fax: 210-458-5947; e-mail: Paul.Cotae@utsa.edu .

Matthew Aguirre is with Crypto Modernization Program Office, Matthew.Aguirre@LACKLAND.AF.MIL , San Antonio, TX, SUA.

In view of this problem, we define a global quantity that is a measure of the total interference in the system, namely Total Generalized Squared Correlation (TGSC). Our definition is based on normal property of Gram matrix assuming that the noise covariance matrix belongs to commuting normal matrices class. We provide a stable distributed iterative method, which allows for each user simultaneously and independent of the others to adjust its signature sequence in order to minimize TGSC. This method is also applicable to the case of overloaded CDMA systems when the number of users is greater then the processing gain and when it is desirable to let some or all users update their signature sequence simultaneously. To accomplish this task the base station needs to send only once for each user the matrix containing the information of all users' signature sequences in the system in presence of the noise.

This paper is organized as follows: In Section II we give a precise model of the multipath uplink Gaussian vector multiple access channels (VMAC) synchronous CDMA systems using Sylvester matrices. Based on this model the definition of the TGSC criterion is introduced and a characterization of joint minimization of bit error rate (BER) and TGSC in multipath is presented. In Section III, we derive the mathematical background in the context of our vector CDMA model. The proposed method of minimizing TGSC is given in Section IV and numerical examples are provided in Section V to illustrate the theoretical developments in the previous sections. Conclusions and future work are presented in Section VI.

II. The Model of Multipath Uplink Synchronous CDMA System

We start considering uplink vector multiple access channel (VMAC) of a multipath single-cell DS-CDMA system with K independent active users and the processing gain N. In the presence of the vector noise process $\boldsymbol{z}$, the received signal in one symbol interval is

$$\boldsymbol{r} = \sum_{k=1}^{K} \sqrt{p_k}\, x_k \boldsymbol{H}_k \boldsymbol{s}_k + \boldsymbol{z} \tag{1}$$

where for user k, p_k is the transmitted power, x_k is the random information symbol with zero mean and variance $E[x_k^2] = 1$. The $N \times K$ users' matrix $\boldsymbol{S} = [\boldsymbol{s}_1, \boldsymbol{s}_2, ..., \boldsymbol{s}_k, ...\boldsymbol{s}_K]$

has unitary signature sequences $\boldsymbol{s}_k$ as N dimensional columns [8], [9], [14], [19]. The spread symbols for user k are passed through the discrete-time channel with the impulse response given by the $N \times 1$ vector

$$\boldsymbol{h}_k = \left[h_k[1], h_k[2], \cdots, h_k[L_k], 0, \cdots, 0\right]^T \tag{2}$$

(by T we mean transpose) where $L_k < N$ represent the number of paths, assumed to be spaced at the chip duration $T_c = T / N$. We also assume a dense multipath profile so the received signal has duration of $N + L_k - 1$ chips, the channel delay spread are small compared with the symbol duration and, hence neglect ISI. The $(N + L_k - 1) \times N$ channel matrix $\boldsymbol{H}_k$ for user k is

$$\boldsymbol{H}_k = \begin{bmatrix} h_k[1] & 0 & \cdots & 0 \\ h_k[2] & h_k[1] & \ddots & \vdots \\ \vdots & h_k[2] & \ddots & 0 \\ h_k[L_k] & \vdots & \ddots & h_k[1] \\ 0 & h_k[L_k] & \ddots & h_k[2] \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & h_k[L_k] \end{bmatrix} \tag{3}$$

Let $\boldsymbol{H}$ be the generalized multipath channel matrix associated with the model given in (1) of the form[1] $\boldsymbol{H} = diag(\boldsymbol{H}_1, \boldsymbol{H}_2, ..., \boldsymbol{H}_K)$ with the same number of paths L_k for each user. The particular case when the multipath channel matrix is of the form $\boldsymbol{H} = \boldsymbol{H}_K$ (i.e. in the point to point transmission when K users are co-located and the received signals of all users go through the same multipath channel matrix) is considered in [24]. For $0 \le i \le N-1$ consider the operation of shifting right the vector (2) by i components as described below

$$D^i \boldsymbol{h}_k = [\underbrace{0,...,0}_{i}, h_k[1], h_k[2], \cdots, h_k[L_k], \underbrace{0, \cdots, 0}_{N-1-i}]^T \tag{4}$$

Using (4) the channel matrix $\boldsymbol{H}_k$ given by (3) becomes Sylvester matrix of order N [4] ($\otimes$ is the Kronecker product acting element wise and I_N denotes the $N \times N$ identity matrix)

$$\begin{aligned} \boldsymbol{H}_k &= [\boldsymbol{h}_k, D\boldsymbol{h}_k, ..., D^i \boldsymbol{h}_k, ..., D^N \boldsymbol{h}_k] \\ &= \left(\begin{bmatrix} 1 & D & \dots & D^N \end{bmatrix} \otimes I_N\right)[\boldsymbol{h}_k, ..., \boldsymbol{h}_k] \end{aligned} \tag{5}$$

[1] We shall denote the diagonal matrix whose main diagonal entries are the same as those of the matrix $\boldsymbol{A}$ as $diag(\boldsymbol{A})$ and the diagonal matrix whose diagonal entries are formed from vector $\boldsymbol{b}$ as $diag(\boldsymbol{b})$. By $diag(\boldsymbol{A}, \boldsymbol{B})$ we mean a block structured matrix $\boldsymbol{A} \oplus \boldsymbol{B} = \begin{pmatrix} \boldsymbol{A} & 0 \\ 0 & \boldsymbol{B} \end{pmatrix}$ [29].

One can prove that the matrix $\boldsymbol{H}_k$ has the following properties:

II.1) $\forall i, j \in \overline{1, K}$ $\boldsymbol{H}_i^T \boldsymbol{H}_j$ is a banded Toeplitz matrix,

II.2) $trace(\boldsymbol{H}_k^T \boldsymbol{H}_k) = L_k \boldsymbol{h}_k^T \boldsymbol{h}_k = L_k$, when $\boldsymbol{h}_k^T \boldsymbol{h}_k = 1$, (6)

II.3) $trace\left[(\boldsymbol{H}_k^T \boldsymbol{H}_k)^{-1}\right] \ge L_k$.

Considering $\boldsymbol{P} = diag(p_1, p_2, ..., p_K)$ and $\boldsymbol{x}^T = (x_1, x_2, ..., x_k)$, then a compact matrix form for the model of uplink DS-CDMA system with multipath given by (1) can be written as

$$\boldsymbol{r} = \boldsymbol{HSP}^{1/2}\boldsymbol{x} + \boldsymbol{z} \tag{7}$$

The noise covariance matrix associated to noise vector $\boldsymbol{z}$ that contains i.i.d circularly symmetric Gaussian noise samples is $\boldsymbol{\Sigma} = E[\boldsymbol{z}\boldsymbol{z}^T]$. In the case of white noise when each vector $\boldsymbol{z}$ has zero mean and the same variance σ^2 then the covariance matrix is $\boldsymbol{\Sigma} = \sigma^2 \boldsymbol{I}_K$. Our approach is more general than [10] were it is assumed that the power spectral densities of thermal noise at all receivers are the same. In the context of VMAC, and considering the case of colored noise, we assume the covariance matrix $\boldsymbol{\Sigma}$ positive definite[2] with its eigenvalues $eig(\boldsymbol{\Sigma}) = \sigma_1^2 \le \sigma_2^2 \le ... \le \sigma_N^2$ as in [13].

Given K, N, $\boldsymbol{S}$ and assuming real signature sequences, the sum capacity of S-CDMA channel per degree of freedom is given by [5], [13]

$$C_{sum} \le \frac{1}{2N} \log_2 \det(\boldsymbol{I}_N + \boldsymbol{\Sigma}^{-1}\boldsymbol{SPS}^T) \tag{8}$$

Let the matrix $\boldsymbol{C}$ of the form $\boldsymbol{C} = \boldsymbol{I}_N + \boldsymbol{\Sigma}^{-1}\boldsymbol{SPS}^T$ and multiplying $\boldsymbol{C}$ left by $\boldsymbol{\Sigma}$ we obtain

$$\boldsymbol{\Sigma C} = \boldsymbol{\Sigma} + \boldsymbol{SPS}^T \tag{9}$$

Consider the determinant of the matrix $\boldsymbol{\Sigma C}$; taking the logarithm of both sizes and multiplying by the constant $\frac{1}{2N}$ we obtain

$$C_{sum} \le \frac{1}{2N} \log_2 \det(\boldsymbol{\Sigma} + \boldsymbol{SPS}^T) - \frac{1}{2N} \log_2 \det(\boldsymbol{\Sigma}) \tag{10}$$

[2] In this paper we assume snapshot analysis for model (7). In N dimensional signal space we assume the eigenvalues $\{\sigma_i^2 \mid i = 1, 2, ..., N\}$ and transmitted powers $p_i, 1 \le i \le K$ are given and fixed. The matrix $\boldsymbol{\Sigma}$ is symmetric and the relationship between diagonal elements and the eigenvalues is precisely characterized by the majorization theory [2].

When we focus on the overloaded S-CDMA systems (loading factor $\beta = K/N > 1$) we need to characterize the effect of the number of users K in the matrix $\boldsymbol{\Sigma} + \boldsymbol{SPS}^T$ used in (9) and (10) which determine sum capacity performance.

Using the Gram matrix approach introduced in [11], [12], [16], and the fact that the matrix $\boldsymbol{SPS}^T$ has the same eigenvalues as the matrix $\boldsymbol{P}^{1/2}\boldsymbol{G}\boldsymbol{P}^{1/2}$, ($\boldsymbol{G} = \boldsymbol{S}^*\boldsymbol{S}$ is the Gram matrix associated with signature sequences), we give the following

Definition: Given a transmitted signature sequence set $\boldsymbol{S_T} = [\boldsymbol{s}_1, \boldsymbol{s}_2, ..., \boldsymbol{s}_K]$ and the power matrix $\boldsymbol{P} = diag(\boldsymbol{p})$ defined by (1) then the total generalized square correlation (TGSC) of received sequences $\boldsymbol{S_R}$ in multipath and in the presence of colored noise $\boldsymbol{z}$ as in (7) is defined by the Frobenius norm of the matrix $\boldsymbol{P^{1/2}G_RP^{1/2}} + \boldsymbol{\Sigma}$ in terms of its eigenvalue[3] ν_i

$$TGSC(\boldsymbol{S_R}) = \left\| \boldsymbol{P^{1/2}G_RP^{1/2}} + \boldsymbol{\Sigma} \right\|_F^2 = \sum_{i=1}^{K} \nu_i^2 \tag{11}$$

This definition includes as particular cases: total correlation (TC) [16], total squared correlation (TSC) [3], [8], [9], [14] and total weighted squared correlation (TWSC) [11], [12], [20], [24], [25], [26]. Under some given conditions in the system is natural to ask that whether a reduction in TGSC always translates into an increase in sum capacity. This problem is the subject of many papers [7-9], [14], [15], [21], [22], and [24] and in here an iterative stable method of reducing TGSC in the presence of multipath and in a controllable number of steps is proposed. The number of steps is linear with the dimensionality of signal space.

In the case of CDMA systems the channel matrix is determined by signature sequences. If we assume matched filter (MF) receivers i.e., then the received power of user k through is receiver filter is

$$\begin{aligned}\tilde{p}_k &= p_k(\boldsymbol{s}_k^*\boldsymbol{H}_k^*\boldsymbol{H}_k\boldsymbol{s}_k) = p_k < \boldsymbol{H}_k\boldsymbol{s}_k, \boldsymbol{H}_k\boldsymbol{s}_k > \\ &= p_k < \tilde{\boldsymbol{s}}_k, \tilde{\boldsymbol{s}}_k >= \alpha_k p_k\end{aligned} \tag{12}$$

where $*$ is Hermitian transpose, $<,>$ is the inner product, $\boldsymbol{S_R} = [\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_K]$, and α_k is the channel state. These channel states can be obtained at the transmitter side by feedback from the receiver, or can be estimated at the transmitter. In the context of Gaussian VMAC the signal to interference ratio for the user k is

[3]Note that in this definition the noise matrix becomes positive semidefinite with the eigenvalues of the form $eig(\boldsymbol{\Sigma}) = [\sigma_1^2, \sigma_2^2, ..., \sigma_N^2, \underbrace{0, ..., 0}_{K-N}]$.

$$SIR_k^{MF} = \gamma_k = \frac{\alpha_k p_k}{\sigma_k^2 + \sum_{\substack{j=1 \\ j \neq k}}^{K} \alpha_j p_j \left| < \tilde{s}_k, \tilde{s}_j > \right|^2} \tag{13}$$

The BER for the k^{th} user is generally a function of the γ_k, and thus the BER $P_b(e\,|\,\alpha_k)$ for a given channel state α_k may be expressed as

$$P_b(e\,|\,\alpha_k) = F(\gamma_k) = F(\alpha_k p_k) \tag{14}$$

where $F(\cdot)$ is a function determined by a specific modulation schemes. Since the random information symbols are independent, the overall BER for a given channel state of $\{\alpha_k, 1 \le k \le K\}$ can be calculated as an arithmetic mean of $P_b(e\,|\,\alpha_k)$

$$P_b(e\,|\,\alpha_1, \alpha_2, ..., \alpha_K) = \frac{1}{K}\sum_{k=1}^{K} P_b(e\,|\,\alpha_k) = \frac{1}{K}\sum_{k=1}^{K} F(\alpha_k p_k) \tag{15}$$

We will show that, under Gram matrix approach, where $\sum_{i=1}^{K} \lambda_i = \Lambda = trace(\boldsymbol{P})$ and in the multipath regime, the objective of minimizing the overall BER given by (15) and the objective of minimizing TGSC criterion in (11) (or increasing sum capacity given by (10)) can lead to different transmitted power allocation $\{p_k, 1 \le k \le K\}$. In order to prove it, consider the Lagrangian function associated with these two objectives of the form

$$J(p_1, p_2, ..., p_K) = f(p_1, p_2, ..., p_K) + \lambda(\sum_{i=1}^{K} p_i - \Lambda) \tag{16}$$

where

$$f(p_1, p_2, ..., p_K) = \frac{1}{K}\sum_{i=1}^{K} Q\left(\sqrt{\gamma_k}\right) = \frac{1}{K}\sum_{i=1}^{K} Q\left(\sqrt{\beta_k p_k}\right) \tag{17}$$

in the case of S-CDMA systems [5],

$$(\,Q(x) = (1/\sqrt{2\pi})\int_x^{\infty} \exp(-t^2/2)dt\,)$$

and recognizing that

$$\beta_k = \alpha_k / (\sigma_k^2 + \sum_{\substack{j=1 \\ j \neq k}}^{K} \alpha_j p_j \left| < \tilde{s}_k, \tilde{s}_j > \right|^2)$$

is independent of transmitted power p_k, and

$$f(p_1, p_2, ..., p_K) = \frac{1}{2N}\sum_{i=1}^{N} \log_2(\sigma_i^2 + \frac{\sum_{k=1}^{K} \beta_k p_k}{N}) \tag{18}$$

in the case of TGSC[4] and in the presence of multipath.

Differentiating (16) with the respect to p_k and setting it to zero, we obtain

$$D(\beta_k, p_k) = -\lambda, \quad 1 \le k \le K \qquad (19)$$

The derivative corresponding to BER minimization is defined as $D_{BER}(\beta_k, p_k) = -\frac{1}{2K\sqrt{2\pi}}\sqrt{\frac{\beta_k}{p_k}}\exp(-\frac{1}{2}\beta_k p_k)$, that is a monotonically increasing function of positive p_k, whereas for sum capacity maximization the derivative

$D_{C_{sum}}(\beta_k, p_k) = (\beta_k / 2N^2 \ln 2)/(\sigma_i^2 + \frac{\sum_{k=1}^{K}\beta_k p_k}{N})$ is a monotonically decreasing function of p_k.

The optimal transmitted powers p_k^* should make the derivative $D(\beta_k, p_k)$ become equal for each user as in (19). According to the behavior of the derivative $D_{BER}(\beta_k, p_k)$ it may be effective to allocate more power to the user with smaller BER in order to reduce overall BER. On the other hand, from the observation on the derivative $D_{C_{sum}}(\beta_k, p_k)$, it can be seen that more power should be allocated to the user with larger β_k (or to user that experiences minimum interference according to (13) as it is also observed in [7], [8], [14]. But this is the well-known behavior of the water-filling policy that characterizes the maximizing sum capacity [23]. In order to compare with the results given in [7-9] the derivation of BER depending of TSC and TWSC is derived in the Appendix.

III. Modified Alternanting Projection Method and Gradient Flow Method

We will present a unified approach of modified alternating projection method and gradient flow [1] method in order to minimize (11). Given two vectors $\boldsymbol{\lambda} = [\lambda_i] \in R^K$ and $\boldsymbol{a} = [a_i] \in R^K$ such that $\boldsymbol{a}$ majorizes $\boldsymbol{\lambda}$, the Schur theorem [2, pp. 218, Theorem B.1] states there exist a Hermitian matrix with eigenvalues $\boldsymbol{\lambda}$ and diagonal entries $\boldsymbol{a}$. Let us define two hyper planes in $R^{K\times K}$ space

$$\Sigma_1(\boldsymbol{a}) = \left\{ \boldsymbol{W} \in R^{K\times K} \,\middle|\, diag(\boldsymbol{W}) = diag(\boldsymbol{a}) \right\} \qquad (20)$$

$$\Sigma_2(\boldsymbol{\Lambda}) = \left\{ \boldsymbol{U^T \Lambda U} \,\middle|\, \boldsymbol{U} \in \boldsymbol{O}(K) \right\} \qquad (21)$$

[4] We assumed that the matrices $\boldsymbol{\Sigma}$ and $\boldsymbol{P}^{1/2}\boldsymbol{G}\boldsymbol{P}^{1/2}$ are joint diagonalized by the same unitary matrix, i.e. they can be written as $\boldsymbol{\Sigma} = \boldsymbol{UDU}^*$ and $\boldsymbol{P}^{1/2}\boldsymbol{G}\boldsymbol{P}^{1/2} = \boldsymbol{UKU}^*$, where $\boldsymbol{U}$ and $\boldsymbol{K}$ are the eigenvalue matrices.

where $\boldsymbol{\Lambda} = diag(\lambda)$ and $\boldsymbol{O}(K)$ is the group of all orthogonal matrices in $R^{K\times K}$. Clearly, by Schur theorem, the shortest distance between $\Sigma_1(\boldsymbol{a})$ and $\Sigma_2(\boldsymbol{\Lambda})$ in the Frobenius norm

$$\min \left\| \boldsymbol{W} - \boldsymbol{V} \right\|_F, \boldsymbol{W} \in \Sigma_1(\boldsymbol{a}), \boldsymbol{V} \in \Sigma_2(\boldsymbol{\Lambda}) \qquad (22)$$

should be zero. Usually alternating projection method is used on convex sets. In the context of our method the set $\Sigma_2(\boldsymbol{\Lambda})$ is not convex and $TGSC(\boldsymbol{S}_R)$ will converge to a sub optimum point.

In modified alternating projection method the main idea is to alternate between Σ_1 and Σ_2 in the following way: From any point $\boldsymbol{W}^{(k)} \in \Sigma_1$, first we find the point $\boldsymbol{V}^{(k)} \in \Sigma_2$ such that $\left\| \boldsymbol{W}^{(k)} - \boldsymbol{V}^{(k)} \right\|_F = dist(\boldsymbol{W}^{(k)}, \Sigma_2)$. Then we find $\boldsymbol{W}^{(k+1)} \in \Sigma_1$ such that $\left\| \boldsymbol{W}^{(k+1)} - \boldsymbol{V}^{(k)} \right\|_F = dist(\Sigma_1, \boldsymbol{V}^{(k)})$. The distance is also measured in Frobenius norm. Since in either step we are minimizing the distance between a point and a set, we have

$$\left\| \boldsymbol{W}^{(k+1)} - \boldsymbol{V}^{(k+1)} \right\|_F^2 \le \left\| \boldsymbol{W}^{(k+1)} - \boldsymbol{V}^{(k)} \right\|_F^2 \le \left\| \boldsymbol{W}^{(k)} - \boldsymbol{V}^{(k)} \right\|_F^2 \qquad (23)$$

The sequence $\left\{ \boldsymbol{W}^{(k)}, \boldsymbol{V}^{(k)} \right\}$ will converge to a stationary point for problem (22). Because $\Sigma_2(\boldsymbol{\Lambda})$ is not a convex set, a stationary point for (22) is not necessarily an intersection point of Σ_1 and Σ_2. The rate of convergence of this method is linear and might not be very efficient. The above method assumed a spectral decomposition of the matrix $\boldsymbol{W}$. In gradient flow method this will not be necessary.

We want to solve a problem similar to (22), i.e. finding the minimum of the following functional $F(\boldsymbol{U})$

$$\min F(\boldsymbol{U}) = \frac{1}{2}\left\| diag(\boldsymbol{U^T}\Lambda\boldsymbol{U}) - diag(\boldsymbol{a}) \right\|_F^2, \boldsymbol{U} \in \mathrm{O}(K) \qquad (24)$$

Again the Schur theorem guarantees that there exists a $\boldsymbol{U}$ matrix at which $F(\boldsymbol{U})$ vanishes. We need the inner product applied to matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ defined in the Frobenius sense

$$\left\langle \boldsymbol{U}, \boldsymbol{V} \right\rangle = \sum_{i=1}^{K}\sum_{j=1}^{K} u_{ij} v_{ij} \qquad (25)$$

Taking the Fréchet derivative of $F(\boldsymbol{U})$ defined in (24) (acting on arbitrary orthogonal matrix $\boldsymbol{Q} \in R^{K\times K}$) and using (25) we have

$$F^{'}(\boldsymbol{U})\boldsymbol{Q} = 2\left\langle diag(\boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{\Lambda U}) - diag(\boldsymbol{a}), diag(\boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{\Lambda Q})\right\rangle$$
$$= 2\left\langle diag(\boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{\Lambda U}) - diag(\boldsymbol{a}), \boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{\Lambda Q}\right\rangle \quad (26)$$
$$= 2\left\langle \boldsymbol{\Lambda U}\left[diag(\boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{\Lambda U}) - diag(\boldsymbol{a})\right], \boldsymbol{Q}\right\rangle$$

Thus the flow gradient $\nabla F(\boldsymbol{U})$ associated to (24) can be written as

$$\nabla F(\boldsymbol{U}) = \boldsymbol{\Lambda U}\left[diag(\boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{\Lambda U}) - diag(\boldsymbol{a})\right] \quad (27)$$

Obviously a necessary condition to have a minimum of (24) at a stationary point $\boldsymbol{U}$ is that the condition $\left[diag(\boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{\Lambda U}) - diag(\boldsymbol{a})\right] = 0$ holds (the case when $\boldsymbol{\Lambda U} = 0$ is trivial). Thus only those matrices $\boldsymbol{X} = \boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{\Lambda U}$ (at which $\left[diag(\boldsymbol{U}^{\boldsymbol{T}}\boldsymbol{\Lambda U}) - diag(\boldsymbol{a})\right] = 0$) are the asymptotically stable equilibrium points for (24). At a stationary point $\boldsymbol{U}$ where $\left[diag(\boldsymbol{U}^{\boldsymbol{T}}\Lambda\boldsymbol{U}) - diag(\boldsymbol{a})\right] \neq 0$ there exists a certain direction along which the functional $F(\boldsymbol{U})$ is increasing. The corresponding equilibrium point $\boldsymbol{X} = \boldsymbol{U}^{\boldsymbol{T}}\Lambda\boldsymbol{U}$ has at least one unstable (repelling) direction. A stationary point $\boldsymbol{U}$ of (22) necessarily corresponds to an equilibrium point $\boldsymbol{X} = \boldsymbol{U}^{\boldsymbol{T}}\Lambda\boldsymbol{U}$ of (24) and vice versa. To show the limitation of the method used in [8], [14] whose convergence was discussed in [18], it is enough to provide an example when $\left[diag(\boldsymbol{U}^{\boldsymbol{T}}\Lambda\boldsymbol{U}) - diag(\boldsymbol{a})\right] \neq 0$ at certain stationary point; such example for TSC is given in Section V, *Experiment 1.*

IV. PROPOSED METHOD

Assume channel matrix $\boldsymbol{H}$ is known both at transmitter and receiver. We minimize TGSC in a multipath channel considering a received vector in $N+L-1$ dimensional space by controlling the transmitted sequence in N dimensional space. We generalize the results obtained in [20] minimizing the Frobenius norm of the matrix $\boldsymbol{P}^{1/2}\boldsymbol{G_R}\boldsymbol{P}^{1/2} + \boldsymbol{\Sigma}$ by controlling its eigenvalues. At each step these eigenvalues are modified using a majorization constraint starting with the maximum eigenvalue. The minimum eigenvalue is always kept constant and is modified only in the last step. In this way, by the proposed method, the norm of the columns of the matrix $\boldsymbol{P}^{1/2}\boldsymbol{G_R}\boldsymbol{P}^{1/2} + \boldsymbol{\Sigma}$ is modified iteratively toward to their global optimum values of maximizing (11). The proposed method is a generalization of uniform good property described in [3] and it achieves the minimization of TGSC in $N+L-2$ steps.

The corresponding spreading signatures sequences are obtained decentralized at each step by using the algorithm given in [24] for uplink multiple cell CDMA systems and it is based on the inverse eigenvalue problem. This algorithm overcomes the limitation of the previous algorithms based on the same idea [6], [13], [26] when they are applied to multiple cells S-CDMA systems. When the algorithm proposed in [24] is applied to a single cell it has the same complexity as the algorithms given in [26].

It is known that so called iterative multiuser water filling method [23] achieves in a fully decentralized way the global optimum of sum capacity, and this coincide with a Nash equilibrium since each user water fills the noise plus interference caused by the others. The method given in [23] is very robust after the first iteration. The method proposed in this paper it is not so robust after the first iteration but it controls exactly the number of steps. Both methods perform optimization in the space of positive semidefinite matrices and do not depend on the initial starting point. However, for a large dimensional signal space and a large number of users it is desirable to let same or all users updating their sequence simultaneously in a finite number of steps.

Given a normalized correlation matrix $\boldsymbol{G_R}$ (the initial point of recursion) in the presence of colored noise matrix $\boldsymbol{\Sigma}$, such that the eigenvalues of TGSC are arranged in decreased order $\nu_i \geq \nu_{i+1} \geq 0$, then $TGSC(\boldsymbol{S_R}) = \sum_{i=1}^{K} \nu_i^2$ and $\sum_{i=1}^{K} \nu_i = p_{tot} + trace(\boldsymbol{\Sigma})$. By Schur theorem for Gram matrix $\boldsymbol{G_R}$ we have the following majorization condition satisfied

$$\underbrace{(1,..,1)}_{K} \prec \left(\frac{K}{N+L-1},...,\frac{K}{N+L-1}, \underbrace{0,...,0}_{K-(N+L-1)}\right) \prec (\nu_1,\nu_2,...,\nu_K) \quad (28)$$

If matrix $\boldsymbol{G_R}$ is chosen at random, obviously $TGSC(\boldsymbol{S_T}) \geq K^2/(N+L-1)$ holds. Our iterative method for minimizing TGSC and obtaining the corresponding WBE sequences is reduced to constructing the matrices $\boldsymbol{W}_i$ and $\boldsymbol{\Sigma}_i$ given by (32) and it consists in the following steps:

1. Input K, N, $\boldsymbol{p} = [p_1 \geq p_2 \geq,...,\geq p_K]$ and noise densities such that $eig(\boldsymbol{\Sigma}) = [\sigma_1^2 \leq \sigma_2^2 \leq,...,\leq \sigma_{N+L-1}^2, 0,...,0]$ (in order to check with the work given in [7], [13]). Determine the oversized users as in [6]. If there are M oversized users use (33). If all users are nonoversized then (28) becomes

$$(p_1+\sigma_1^2, p_2+\sigma_2^2,..., p_K+\sigma_K^2) \prec$$
$$\prec \left(\frac{p_{tot}+trace(\Sigma)}{N+L-1}, \frac{p_{tot}+trace(\Sigma)}{N+L-1},..., \frac{p_{tot}+trace(\Sigma)}{N+L-1}, 0,...,0\right)$$
$$\prec (\nu_1,\nu_2,...,\nu_K) \quad (29)$$

2. Given a random weighted Gram matrix $\boldsymbol{W}$ of order $K \times K$ and rank N calculate $eig(\boldsymbol{W})$. Algorithms to generate random weighted Gram matrices are well known; see for example "gallery" Higham test matrices, provided in

Matlab. Find a permutation matrix $\boldsymbol{Q}$ such that the matrix $\boldsymbol{W}_0 = \boldsymbol{Q}^T \boldsymbol{W} \boldsymbol{Q}$ has eigenvalues $eig(\boldsymbol{W}_0) = \boldsymbol{\Lambda}_0 = (\lambda_1, \lambda_2, ..., \lambda_K)$. Find orthogonal matrix $\boldsymbol{U}_0$ satisfying $\boldsymbol{W}_0 = \boldsymbol{U}_0^T diag(\boldsymbol{\Lambda}_0) \boldsymbol{U}_0$ using the Algorithm given in [24]. Assuming that $\boldsymbol{W}$ and $\boldsymbol{\Sigma}$ commute the same procedure is valid for $\boldsymbol{\Sigma}$ so we get $\boldsymbol{\Sigma}_0 = \boldsymbol{U}_0^T diag(\boldsymbol{\sigma}_0) \boldsymbol{U}_0$ where

$$\boldsymbol{\sigma}_0 = eig(\boldsymbol{\Sigma}_0) = [\sigma_1^2, \sigma_2^2, ..., \sigma_{N+L-1}^2, 0, .., 0].$$

Let $\boldsymbol{v}_0 = \boldsymbol{\Lambda}_0 + \boldsymbol{\sigma}_0$.

3. For $1 \le n \le N + L - 2$ construct majorization eigenvalues sets

$$\boldsymbol{v}_n = \begin{pmatrix} \underbrace{\frac{p_{tot} + trace(\Sigma_0)}{N+L-1}, ..., \frac{p_{tot} + trace(\Sigma_0)}{N+L-1}}_{n}, \\ v_1 + v_2 + ... + v_{n+1} - n\frac{p_{tot} + trace(\Sigma_0)}{N+L-1}, \\ v_{n+2}, ..., v_{N+L-1}, \underbrace{0, ..., 0}_{K-(N+L-1)} \end{pmatrix} \quad (30)$$

At each step the old set of eigenvalues is replaced by a new one using majorization constraint $\boldsymbol{v}_{n+1} \prec \boldsymbol{v}_n$. In the last step for $n = N + L - 2$ we have

$$\boldsymbol{v}_{L+N-1} = \left(\frac{p_{tot} + trace(\Sigma_0)}{N+L-1}, ..., \frac{p_{tot} + trace(\Sigma_0)}{N+L-1}, 0, ..., 0 \right) \quad (31)$$

4. Construct matrices

$$\boldsymbol{W}_i + \boldsymbol{\Sigma}_i = \boldsymbol{U}_i^T \left[\boldsymbol{U}_0 (\boldsymbol{W}_0 + \boldsymbol{\Sigma}_0) \boldsymbol{U}_0^T - diag(\boldsymbol{v}_0 - \boldsymbol{v}_i) \right] \boldsymbol{U}_i \quad (32)$$

If there are M oversized users in system then majorization constraint given by (26) becomes

$$\begin{aligned} &(p_1 + \frac{trace(\boldsymbol{\Sigma})}{N+L-1}, p_2 + \frac{trace(\boldsymbol{\Sigma})}{N+L-1}, ..., p_K + \frac{trace(\boldsymbol{\Sigma})}{N+L-1}) \\ &\prec (p_1 + \sigma_1^2, p_2 + \sigma_2^2, ..., p_M + \sigma_M^2, \underbrace{v, ..., v}_{N+L-1-M}, \underbrace{0, ..., 0}_{K-(N+L-1)}) \quad (33) \\ &\prec (v_1, v_2, ..., v_K) \end{aligned}$$

where

$$v = \frac{\sum_{l=M+1}^{K} (p_l + \sigma_l^2)}{N+L-1-M} \quad (34)$$

A oversized user will use alone the signal dimension in signal space.

For $1 \le l \le N + L - 1 - M$ (33) becomes

$$\boldsymbol{\mu}_l = (p_1 + \sigma_1^2, ..., p_M + \sigma_M^2, v_1 + v_2 + ... + v_{l+1} - \sum_{i=1}^{l} (p_i + \sigma_i^2), \underbrace{v, ..., v}_{N+L-1-M}, \underbrace{0, ..., 0}_{K-(N+L-1)}) \quad (35)$$

and repeat steps 3) and 4). In the last step we have

$$\boldsymbol{\mu}_{L+N-1} = (p_1, ..., p_M, \underbrace{v, ..., v}_{N+L-1-M}, \underbrace{0, ..., 0}_{K-(N+L-1)}) \quad (36)$$

which is the desired eigenvalue set.

V. NUMRICAL RESULTS

Experiment 1. We start with a numerical example when the condition $diag(\boldsymbol{U}^T \Lambda \boldsymbol{U}) - diag(\boldsymbol{a}) \neq 0$ is violated. This condition is equivalent with the relation [(25), 14] and repeated here:

$$\Delta = Trace_{before}[(\boldsymbol{R}_k + \boldsymbol{s}_k \boldsymbol{s}_k^T)^2] - Trace_{after}[(\boldsymbol{R}_k + \boldsymbol{x}\boldsymbol{x}^T)^2] \ge 0.$$

By this numerical example we show that algorithms for maximizing sum capacity or minimizing interference avoidance [14] are not identically and, in general neither they are disjoint. Minimizing the TGSC (TSC or TWSC) does not imply, in general, the convergence of the signature set $\boldsymbol{S}$ to the optimal sequences. Vice versa, giving a signature set characterizing the CDMA system at a given channel state, using a method of convergence of this signature set to optimal sequences (possible in a finite number of steps) it is not necessarily equivalent with the decreasing of TGSC (TSC or TWSC) at each step. Consider $K = 4, N = 3$.

The method of minimizing TSC described in section IV requires only 2 steps. Random matrices $\boldsymbol{G}$ and $\boldsymbol{G}_0$ are given below. It easy to verify that $TSC(\boldsymbol{G}) = TSC(\boldsymbol{G}_0) = 6.6859$, $eig(\boldsymbol{G}) = [0 \ 0.4060 \ 1.6200 \ 1.9740]$, and $eig(\boldsymbol{G}_0) = [1.9740 \ 1.6200 \ 0.4060 \ 0]$.

$$\boldsymbol{G} = \begin{bmatrix} 1.0000 & 0.4337 & 0.2636 & 0.7438 \\ 0.4337 & 1.0000 & 0.5732 & 0.1398 \\ 0.2636 & 0.5732 & 1.0000 & -0.4291 \\ 0.7438 & 0.1398 & -0.4291 & 1.0000 \end{bmatrix}$$

After the first iteration we got $TSC(\boldsymbol{G}_1) = 7.0534 > TSC(\boldsymbol{G}_0)$. In the last iteration we get $TSC(\boldsymbol{G}_2) = 5.333 = TSC(WBE) = 16/3$.

$$\boldsymbol{G}_0 = \begin{bmatrix} 1.0000 & -0.0683 & -0.1140 & 0.9740 \\ -0.0683 & 1.0000 & 0.5991 & 0.0683 \\ -0.1140 & 0.5991 & 1.0000 & 0.1140 \\ 0.9740 & 0.0683 & 0.1140 & 1.0000 \end{bmatrix}$$

It easy to prove that, in general, in order to have $TSC(\boldsymbol{G}_1) > TSC(\boldsymbol{G}_0)$ after the first iteration, it is sufficient that $K/N \notin [\lambda_1, \lambda_2]$.

Experiment 2. Consider $K = 8, N = 5, L = 2$ in order comparing with the results given in [7, Fig.1]. The noise variances are

$$\boldsymbol{\sigma} = [0.023 \quad 0.947 \quad 1 \quad 1.005 \quad 1.010 \quad 1.015 \quad 0 \quad 0 \quad 0 \quad 0]$$

and all users have unitary power. We started with a random weighted Gram matrix and after permutation the following eigenvalue vector gives the eigenvalues of initial point of iterations:

$$\Lambda_0 = [3.673 \quad 2.426 \quad 2.316 \quad 2.305 \quad 2.150 \quad 2.130 \quad 0 \quad 0].$$

The results are plotted in Fig.1 when we compared the proposed method with TSC minimization algorithm suggested in [7], [8]. In only 5 steps (updates) the proposed method in Section IV produces an optimal spreading signature set that reaches the maximum of the sum capacity.

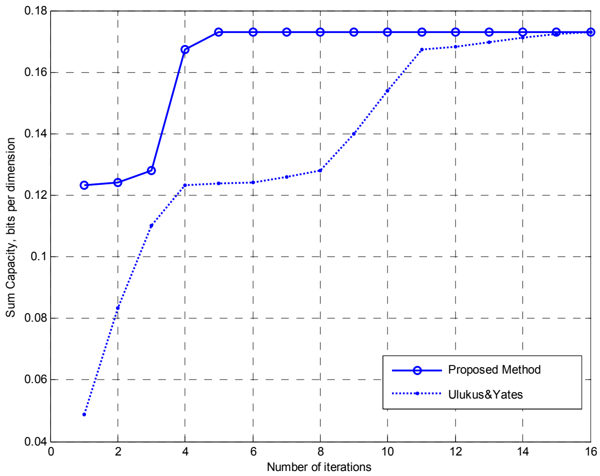

Fig.1. Sum capacity maximization in function of number of updates necessary for K=8 users with N=5 signal processing gain, and L=2 multipath

The proposed method is convergent and stable for any channel realization in contrast with the results of TSC minimizing algorithm for Channel B [7, Fig.2]. We also compared our method with the algorithm proposed in [8] from BER perspective. The results are plotted in Fig.2 where we used the results deduced in the Appendix for both procedures. The improvements of our methods are substantial for minimizing TSC in the range $TSC \in [10.66 \quad 12]$. This result is expected since from Fig.1, for $TSC = 12$, by using the proposed method the optimal signature set is only a step away from the Welch bound [1] while the algorithm proposed in [7],[8] requires 11 additional updates. The main advantage of the proposed method is relevant in the case of the oversized users. Such results are not reported in [7]. Consider the following case where the powers of users are $p_1 = 9, p_2 = 7$, $p_3 = ... = p_8 = 1$ and the noise variances are the same. There are two oversized users and method requires only 3 steps. The results are summarized in Table 1.

TABLE I

8 USERS, 2 OVERSIZED USERS IN 6 DIMENSIONS

Iteration	0	1	2	3
TGSC($\mathbf{S_R}$)	179.8551	180.5177	180.0418	175.5419
TSC($\mathbf{S_R}$)	131.0842	130.9981	130.2137	129.0000

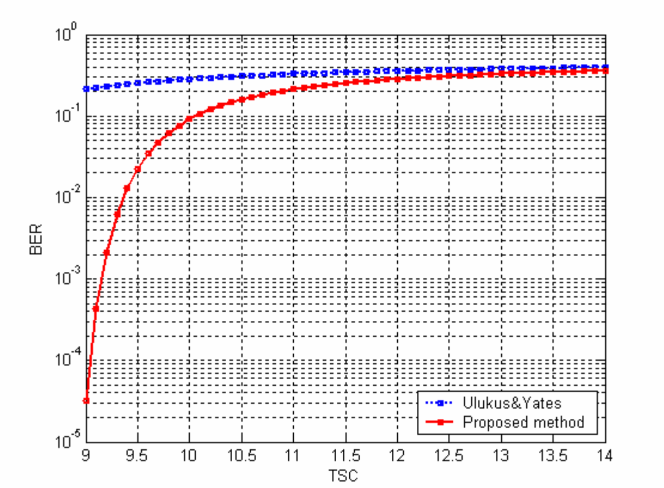

Fig.2 Bit error probability versus TSC for K=8,N=5, L=2

Experiment 3. In this experiment we compare the sum capacity maximization obtained by the proposed method with the sum capacity of parallel Gaussian channels (the maximum sum of rates per unit channel at which all the information can be transmitted in each of the channels reliably). Considering the same data as in *Experiment 2*, for unitary power we can apply water filing for three users. The water level in this numerical example is $.5(trace(\boldsymbol{P}) + \sum_{i=1}^{3} \sigma_i^2)$ and the results are plotted in Fig.3.

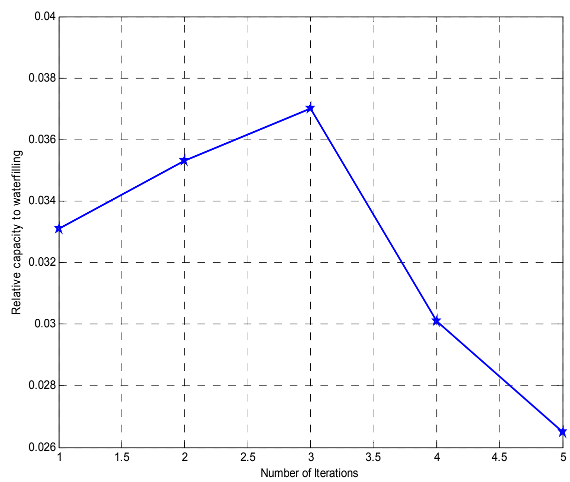

Fig.3 Relative capacity versus water filling method

It is more relevant in this figure that it is not necessary to increase the sum capacity after each step as it was required in previous works [8], [9], [13], [25]. For example, in the first three steps the sum capacity is decreasing and after that is increasing to its maximum value in the next two steps.

VI. CONCLUSIONS AND FUTURE WORK

In this paper we have focused on symbol-synchronous CDMA systems in the presence of colored noise. The extension of our results to the asynchronous situation and considering colored noise is interesting and also an important open problem. Our current efforts are directed towards solving this important open question. The results obtained in this paper are valid for one base station.

The extension to multiple base stations is a challenging task. Using a Gram matrix approach we defined Total Generalized Squared Correlation through the eigenvalues on weighted Frobenius norm of the received sequences in multipath channels and colored noise.

Controlling these eigenvalues we provided an iterative distributed method for minimizing TGSC (TSC, TWSC) and examples are given for oversized and nonoversized users in the context of overloaded CDMA systems.

APPENDIX

It is instructive to consider first a S-CDMA system using WBE signature sequences and for the sake of simplicity we also consider unitary powers. We will derive BER of this system and after that the generalization for non-unitary power sequences (or generalized WBE sequences) follows. At the transmitter side, the total interference in the system is defined as $I = \sum_{\substack{i,j=1 \\ i \neq j}}^{K} \left| < \boldsymbol{s}_i, \boldsymbol{s}_j > \right|^2$ and it is minimized in the case of WBE sequences to

$$I_{\min} = \sum_{\substack{i=1 \\ i \neq j}}^{K} \sum_{j=1}^{K} \left| < \boldsymbol{s}_i, \boldsymbol{s}_j > \right|^2 = \frac{K^2}{N} - K \qquad (37)$$

and the single interference $I(k) = \sum_{\substack{j=1 \\ j \neq k}}^{K} \left| < s_k, s_j > \right|^2$ for the user k is minimized to

$$I_{\min}(k) = \sum_{\substack{j=1 \\ j \neq k}}^{K} \left| < \boldsymbol{s}_k, \boldsymbol{s}_j > \right|^2 = \frac{\frac{K^2}{N} - K}{K} = \frac{K}{N} - 1 \qquad (38)$$

and it is the same for all users in the system. This is so called uniform good property [3]. Note that in general, the values of the particular interferences $I(i)$ and $I(j)$ for different users i and j are not equal since $\left| < \boldsymbol{s}_i, \boldsymbol{s}_j > \right|^2$ are different $\forall i, j = 1, ..., K, i \neq j$. However, the expected value of the single user interference can be obtained by

$$E\left[I(k)\right] = \frac{\sum_{k \neq j}^{K} \left| < \boldsymbol{s}_k, \boldsymbol{s}_j > \right|^2}{K-1} = \frac{(K-N)}{N(K-1)} \qquad (39)$$

Considering (37) then the BER for unitary power valued WBE sequences (real or complex) is the same for each user k and in the case of overloaded systems is reduced to

$$\Pr(\hat{x}_k \neq x_k) = Q[(\sigma^2 + I_{\min}(k))^{-1/2}] = Q[(\sigma^2 + \frac{K}{N} - 1)^{-1/2}] \qquad (40)$$

Consider now the case when all users have different powers and no user is oversized. The total power in the system is $p_{tot} = \sum_{i=1}^{K} p_i$. The Gram matrix associated has the optimum eigenvalue distribution vector given by $(\frac{p_{tot}}{N}, ..., \frac{p_{tot}}{N}, \underbrace{0, ...0}_{K-N})$ [11], [20] and (38) becomes:

$$I_{\min} = \sum_{i=1}^{K} \sum_{j=1}^{K} p_i p_j \left| < s_i, s_j > \right|^2 = \frac{p_{tot}^2}{N} - \sum_{i=1}^{K} p_i^2 \qquad (41)$$

In order to obtain BER we want the above expression to be expressed in term of TWSC. Let's consider the following majorization relation that characterizes the minimum TWSC in this particular case

$$(p_1, p_2, ..., p_K) \prec (\underbrace{\frac{p_{tot}}{N}, \frac{p_{tot}}{N}, ..., \frac{p_{tot}}{N}}_{N}, \underbrace{0, ..., 0}_{K-N}) \qquad (42)$$

Multiplying both terms by K and simplifying by p_{tot} we obtain

$$\left(\frac{Kp_1}{p_{tot}}, \frac{Kp_2}{p_{tot}}, ..., \frac{Kp_K}{p_{tot}}\right) \prec (\underbrace{\frac{K}{N}, \frac{K}{N}, ..., \frac{K}{N}}_{N}, \underbrace{0, ..., 0}_{K-N}) \qquad (43)$$

which correspond to WBE sequences having uniform good property (each column of Gram matrix has the same norm and it is associated to the interference of each user). Thus

$$I_{\min}(k) = \frac{p_{tot}^2}{KN} - p_k^2 = \frac{TWSC}{K} - p_k^2 \qquad (44)$$

and it is the same for each user. Using (40) BER becomes:

$$\Pr(\hat{x}_k \neq x_k) = Q\left[(\sigma^2 + \frac{TWSC}{K} - p_k^2)^{-1/2}\right] \qquad (45)$$

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Paul Cotae was born on June 21, 1955 in Falticeni, Romania. He received Dipl. Ing. and M.S. degrees in communication and electronic engineering in 1980 from Technical University of Iassy and Ph.D. degree in telecommunications from Politechnica University of Bucharest, Romania in 1992.

Since 1984, he has been with the Department of Electrical Engineering, Technical University of Iassy, where he conducted research and teaching in the area of digital communications as a Full Professor at the same department. From 1994 to 1998 he spent four years in the USA at University of Colorado at Colorado Springs and Boulder as a Fulbright Scholar and Visiting Associate Professor doing research and teaching in ECE department and APPM department. He served also as a consultant to Navsys Corp., Colorado Springs, USA in 1997. Now he joined the University of Texas at San Antonio. His current research interests include multiple access, modulation and coding, mobile communications, and digital communication systems. He has authored or coauthored more than 80 papers in these areas and 4 books.

Dr. Paul Cotae serves as an Associate Editor for IEEE COMMUNICATION LETTERS journal and he has been on Technical Program Committee and Session chair of the IEEE Conferences GLOBECOM (2003-2006), VTC Spring 2005 and ICC 2005 and 2006. He is member of HKN (Eta Kappa Nu), ASEE and SIAM.

Matt Aguirre was born in SanAntonio 1978. He is a PhD. Candidate at The University of Texas at San Antonio (UTSA). He graduated with an electrical engineering degree from Texas A&M University in College Station in December 2001. He completed his Master Degree at UTSA in Aug.2005. He is currently working in wireless communications focusing on the next generation CDMA systems.