Adaptive Turbo-Coded Hybrid-ARQ in OFDM Systems over Gaussian and Fading Channels

Kingsley Oteng-Amoako and Saeid Nooshabadi

Abstract—In this paper, an analytical approach for spectral efficiency maximization of coded wideband transmissions is presented based on OFDM. The approach exploits Type-III Hybrid-ARQ, enabling all sub-carriers to be employed in codeword transmission regardless of the sub-carrier conditions. The effects of imperfect sub-channel estimation are characterized and compensated for during code rate and signal constellation optimization. The results of the paper highlight that by independently adapting the code rate and signal constellation to individual OFDM sub-carriers based on an estimated sub-carrier CSI, the overall spectral efficiency of the system is maximized.

Index Terms — Hybrid-ARQ, AMC, adaptive, fading, diversity combining and turbo codes.

I. INTRODUCTION

In fading channels, an increasing method employed in wideband communication is Orthogonal-Frequency-Division-Multiplexing (OFDM) [1] [2]. OFDM in frequency-selective channels effectively presents a set of sub-carriers each with a channel impulse response corresponding to a flat fading channel behavior in nature [2]. Hence, Forward-Error-Correction (FEC) codes employed in flat fading scenarios can be employed in an OFDM system. Turbo codes as a FEC scheme, provide close to capacity performance through iterative decoding of Recursive Systematic Convolutional (RSC) codes [3]. The Turbo encoder is a parallel concatenation of multiple statistically independent RSC codes. The statistical independence between encoders, is provided through separation of RSC encoders by random interleavers, an encoding structure that enhances the overall performance of the iterative decoding scheme.

Hybrid Automatic-Repeat-reQuest (Hybrid-ARQ) combines the flexibility of an ARQ with the error correction capabilities of FEC to provide significant coding and energy gains by combining multiple transmit attempts across a communication channel [4]. The combination of Turbo Hybrid-ARQ with OFDM, provides significant bandwidth at close to capacity rates of the channel. This paper proposes an Adaptive Modulation and Coding (AMC) approach with incremental redundancy employed independently within each sub-carrier in order to maximize the spectral efficiency [5] [6] [7]. The scheme proposes Type-III Hybrid-ARQ with OFDM in order to utilize the complete set of sub-carriers.

In the proposed scheme, Type-III Hybrid-ARQ is also employed within each sub-carrier in order to minimize the Bit Error Rate (BER) over multiple transmit attempts and thus enable all sub-carriers to be used regardless of the Channel-State Information (CSI) [8], [9]. Type-III Hybrid-ARQ combines self-decodable transmissions into a single low rate codeword [10]. The aim of the proposed scheme of combining Type-III Hybrid-ARQ with OFDM across a given sub-carrier \( k \), is to select on the \( t \)-th instantaneous transmit attempt \( R_{k,t}^* \) and \( M_{k,t}^* \), for \( t = 0, 1, \ldots, l \) such that the throughput of individual sub-carriers \( \eta_{k,t}^* \) is maximized whilst not exceeding the target-BER. Given the \( t \)-th transmission in a sub-carrier of an OFDM scheme of a total \( \tau \) sub-carriers, the optimization by selecting \( (R_{k,t}^*, M_{k,t}^*) \) is a \( \tau \) discrete maximization expressed as

\[
\max_{(R_{k,t}^*, M_{k,t}^*)} \arg \left( \eta_{k,t}^* \right)
\]

where \( R_{k,t}^* \) and \( M_{k,t}^* \) are the code rate and signal constellation respectively of the \( t \)-th transmission of the \( \hat{k} \)-th sub-carrier given a system subject to the constraints of power \( S \) and the BER \( P_\text{e} \).

The remainder of the paper is organized as follows. In Section II, the OFDM system description along with channel capacity expressions are presented. In Section III, a Markovian based channel state quantization expression is given. In Section IV, utilizing the Markovian description the proposed adaptation algorithm is described. In Section V, simulation results are presented. The conclusion is given in Section VI.

II. SYSTEM MODEL

The system model in Figure 1 consists of the Turbo coded OFDM transmitter and receiver. The data source feeds a binary encoder. The blocks of the data are subsequently divided into \( \tau \) data blocks and encoded into \( \tau \) separate transmit blocks.

The \( t \)-th transmit block is initially encoded by a Cyclic-Redundancy-Check (CRC) code. The output of the CRC encoder is fed into a Rate Compatible Punctured Turbo (RCPT) encoder consisting of parallel concatenated RSC codes separated by an interleaver. The resulting rate \( 1/3 \) code of length \( n_t \), is buffered before being punctured [8], [11]–[14]. The set of generated codewords for the set of sub-carriers, \( c \) can be represented as a \( (\tau \times 1) \) matrix, \( c = [c_0, c_1, \ldots, c_{\tau - 1}]^T \). The resulting codewords are modulated based on either M-ary PSK or M-ary QAM, such that each transmitted symbol has Q-bits. The OFDM scheme results in an orthogonal channel response unique to each sub-carrier that is applied to respective
codeword blocks by passage of individual sub-carrers through an Inverse Discrete Fourier Transform (IDFT). The IDFT of the encoded symbols on the $k$-th sub-carrier is given by

$$x_{k,q} = \frac{1}{\sqrt{Q}} \sum_{h=0}^{Q-1} c_{h,k} e^{j2\pi \frac{h}{Q} q}$$

where $c_{h,k}$ is the encoded bit, $x_{k,q}$ is the $q$-th bit on the $k$-th sub-carrier and $Q$ is the bit size of the transmitted symbol.

A. Channel Estimation And Imperfect Channel State Information

In OFDM, pilot symbols are employed in transmitted frames in order to estimate the channel state in addition to determining the carrier frequency offset. OFDM can employ a two-dimensional pilot-assisted modulation in order to determine the channel state [15], [16]. Thus given the probability density function of both an ideal CSI and the estimated CSI, the adaptive scheme can be employed in an imperfect CSI scenario.

If the channel is assumed modeled by the classical Jakes channel model, the normalized power spectrum is given as

$$h(f) = \begin{cases} \frac{\pi f_d}{\sqrt{1-(\frac{f}{f_d})^2}} & \text{if} \mid f \mid < f_d \\ 0 & \text{otherwise} \end{cases}$$

where $f_d$ is the Doppler frequency. In estimating the channel at the receiver, Weiner filtering is employed

$$\hat{h}_{k,(t=0)} = Wp = R_{hp}R_{pp}^{-1}p$$

where $Wp$ is the linear minimum mean square error (LMMSE), $p$ is a vector of back-rotated observations at different pilot positions, $R_{hp}$ is the cross co-variance between the estimated channel attenuation $\hat{h}_{k,t}$ and the observations $p$, and $R_{pp}$ is the auto-covariance matrix of observations. If the channel remains constant, this estimate can be assumed accurate and used for the detection of the symbols during the whole information frame.

However, the channel is rapidly time varying in nature. In order to track the divergence of the channel away from the initial estimate, the Per-Survivor-Processing (PSP) technique is exploited in this paper [17]. The divergence tracking algorithm is written as

$$\hat{e}_{k,(t+1)} = \hat{h}_{k,t} + \beta \hat{e}_{k,(t-(t+1))}$$

where $\hat{h}_{k,t}$ is the transmitted bit at a time $t$ on the $k$-th sub-carrier, $\beta$ is an adaptation parameter selected as a compromise between the speed of convergence and a stable estimate. The expression $\hat{e}_{k,(t-(t+1))}$ is estimated from the following equation

$$\hat{e}_{k,(t-(t+1))} = \hat{y}_{k,t+1} - \hat{h}_{k,t}^T \hat{\tilde{y}}_{k,(t+1)}$$

where $\hat{y}_{k,t}$ is the mean of the received bit, at a time $t$ on the $k$-th sub-carrier.

Assuming that the channel estimate can be considered complex Gaussian, the correlation coefficient is given by [18],

$$\delta = I_0(2\pi f_d T_d)$$

where $I_0$ is the zeroth order Bessel function and $T_d$ the absolute magnitude difference in time between channel estimation and the instance at which the codeword is transmitted.

Given the instantaneous and estimated sub-carriers SNR characteristic CSI, $J_{\gamma}$ and $J_{\gamma'}$ respectively, the average throughput obtained across all sub-carriers $\bar{\eta}_t$ is defined as

$$\bar{\eta}_t = E_{J_{\gamma},J_{\gamma}'} \left\{ \frac{1}{T} \sum_{i=0}^{T-1} \eta_{i,t} \right\}$$

As the estimated CSI, $J_{\gamma'}$, differs from that of the actual CSI due to the error effects of an imprecise estimation, it leads to a degradation in the obtainable $\bar{\eta}_t$. The empirical Mean Square Error (MSE) of an imperfect CSI on the $k$-th carrier is given as

$$e_k = E \left\{ \frac{1}{T} \sum_{i=0}^{T-1} |J'_{k} - J'_{k}|^2 \right\}$$

where $J_k$ and $J'_{k}$ are the instantaneous and estimated SNR of the $k$-th sub-carrier respectively, such that $J_k = J'_{k} - e_k$. Thus provided that the effects of imperfect CSI can be estimated to determine $e_k$, the adaptive scheme can accordingly select parameters to counter the effects. From (9) and (7), the corresponding error in a CSI estimate considering the effects of delay and Doppler spread, is expressed as $e_k = 2 - 2\delta$. Correspondingly, the target BER in the presence of imperfect CSI is given as $P_0' = P_0((1 - e_k)H_k)$ [16].
B. Receiver

In the analysis, assume that the OFDM signal is designed such that the effects of inter-symbol interference and the Inter-Carrier Interference (ICI) can be ignored. In order to achieve this, the cyclic prefix (CP) appended to the beginning of each block of transmit symbols is assumed larger than the channel memory. It is assumed that the channel state remains constant during the transmission of an OFDM symbol, with no contributing effects of Inter-Symbol Interference (ISI) or ICI at the receiver. The received OFDM signal is demapped by passage through a Discrete Fourier transform (DFT) before demodulation. The output of the decoder is fed into a buffer for codeword ordering. In the event of a decoding error, a NAK is generated and the codeword is buffered for later code combining, otherwise a positive ACK is sent to the receiver.

Thus the received symbol after demodulation of the DFT output and removal of the CP in the \(k\)-th sub-carrier can be expressed as [1]
\[
y(\hat{k}, t) = H(\hat{k}, t)x(\hat{k}, t) + n(\hat{k}, t)
\]
(10)
where \(y(\hat{k}, t)\) is the received signal of \(Q\) bits on the \(\hat{k}\)-th sub-carrier, \(H(\hat{k}, t)\) is the time response, \(x(\hat{k}, t)\) is the generated OFDM signal and \(n(\hat{k}, t)\) is the zero-mean Gaussian noise with \(E\{\{n\}\} = N_0\). Assuming that the channel remains static during the transmission of a codeword, the ensemble received signal over \(\tau\) sub-carriers can be expressed as
\[
y = Hx + n
\]
(11)
such that the sub-carrier SNR characteristics, \(J_{\gamma}\), of \(\tau\) sub-carriers is given by
\[
J_{\gamma} = \frac{E_s}{N_0} \begin{bmatrix}
|H_0|^2 & 0 & \cdots & 0 \\
0 & |H_1|^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & |H_{\tau-1}|^2
\end{bmatrix} \cdot
\]
where \(E_s\) is the single sided energy. The capacity over \(\tau\) sub-carriers of the transmitter is then given by [18],
\[
C = (B_{\omega,\hat{k}}) \sum_{\hat{k}=0}^{\tau} \log_2 \left(1 + J_{\hat{k}}\right)
\]
(12)
where \(B_{\omega,\hat{k}}\) is the available bandwidth per sub-carrier and \(J_{\hat{k}}\) is the estimated CSI characteristic of a sub-carrier.

C. Channel State Quantization

In order to assign a signal constellation and code rate to each sub-channel, the range of possible SNRs are quantized into multiple channel state intervals. A channel state interval, is characterized by having a mean SNR. In the analysis, the range of possible SNRs is divided into \(\kappa\) groups, resulting in \(\kappa\) channel quantization intervals. The resulting \(i\)-th quantization state interval, denoted \(\Gamma_i\) refers to a channel with a mean SNR,
\[
\mathcal{J}_k = \int_a^{\infty} J_k^2 \rho(J_k) dJ_k
\]
(13)
where \(\rho(J_k)\) is the probability density function of the sub-carrier. Thus \(\mathcal{J}_k\) is a random valued variable of probability density function \(\rho(\mathcal{J}_k)\) and cumulative density function \(F(\mathcal{J}_k)\). The channel quantization interval \(\Gamma_i\) is determined prior to transmission and the range of quantization intervals remains constant over the period of a transmitted codeword. Given that the channel consists of \(\tau\) sub-carriers, the complete characterization of the channel is obtained by determining \(\Gamma_i\) for each of the \(\tau\) sub-carriers at time \(t\). Thus at a given time interval, the complete characterization of the channel is expressed as
\[
\Gamma = [\Gamma(0,0), \ldots, \Gamma(\kappa-1,0), \ldots, \Gamma(0,\bar{k}), \ldots, \Gamma(\kappa-1,\bar{k}), \ldots, \\
\Gamma(0,\tau-1), \ldots, \Gamma(\kappa-1,\tau-1)]
\]
(14)
where \(\Gamma(i,\bar{k})\) is the sub-channel interval for the \(i\)-th quantization state of the \(\bar{k}\)-th sub-carrier. The system analysis assumes that each sub-carrier is subject to the same range in SNRs, correspondingly the subscript \(\bar{k}\) can be dropped and \(\Gamma_i\) used to refer to the \(i\)-th quantization interval of a given sub-channel.

III. Optimization Problem

The goal of the adaptation algorithm is to maximize the throughput of the overall system by maximizing the number of bits sent error-free on a sub-carrier during each transmit instant. The use of OFDM allows the transmitted sequence to minimize the effects of fading across a wide-band channel and thus maximize the overall throughput of the system. Given that the throughput per sub-carrier \(\eta\), the optimization problem can be expressed as
\[
\int_{J_k=0}^{\infty} \eta(J_k) \rho(J_k) dJ_k
\]
(15)
Given that the upper-bound in throughput can be expressed as,
\[
\max \eta(J_k) = R(J_k) m(J_k)
\]
(16)

hence, the transmit maximization problem is a joint-maximization of \(R\) and \(2^m = M\) across the channel quantization state of a sub-carrier \(\Gamma_i\). Thus the required mean throughput is given by
\[
\max \left[ \int_{J_k=0}^{\infty} R(J_k) m(J_k) \rho(J_k) dJ_k \right]
\]
(17)
Communication systems generally operate with an expectation that received codeword will be lower bounded by a maximum amount of error, termed the target error rate of the system. Thus an additional criteria in the design of the system is the target BER \(P_o\).

IV. Code Rate and Signal Constellation Optimization

The goal of the following optimization approach is to optimize the code rate and signalling constellation on each sub-carrier separately, such that the spectral efficiency is maximized on the first transmit attempt in the Hybrid-ARQ scheme.
A. Constellation adaptation

In selecting the constellation parameter for the Turbo coded system, it is assumed that the code has close to ergodic capacity performance. Thus, \( P_\phi \) given an AWGN channel is given by Equation (18) [19], [20]

\[
P_b \leq \left( 2 - 2^{1-m/2} \right) \frac{N_{\text{eff,free}}}{K} \exp \left( -\frac{1.5d_{\text{min}}}{2^{m} - 1} \gamma \right) \tag{18}
\]

where \( N_{\text{eff,free}} \) is the multiplicity of words generating an effective free distance and \( d_{\text{min}} \) is the minimum distance. Hence the signal constellation required on the \( k \) sub-carrier is given as

\[
m(\gamma) = \log_2 \left( 1 - \frac{1.5d_{\text{min}}}{\log_2 \left( \frac{K\gamma}{N_m} \right)} \frac{S(\gamma)}{S} \right), \tag{19}
\]

The transmitter in the system, has the option of not sending any codewords on a sub-carrier if the required code rate results in a channel outage [21]. In our adaptation scheme, codewords are transmitted under all conditions and hence during optimization, the signalling constellation is assumed lower bounded as \( M = 4 \) (\( m=2 \)).

Type-III combining is employed on retransmit attempts of codewords in order to minimize the effects of any residual errors. In the event that the transmitted codeword is received in error despite the parameter optimization approach, the receiver code combines the multiple transmit attempts based on a Type-III combining scheme. This approach of combining AMC with code combining, allows all sub-carriers to be used for transmission thus enabling higher throughput levels than previously suggested schemes [22].

B. Code Rate Adaptation

The problem of obtaining the optimal code rate to achieve ergodic capacity is now addressed. Given the quantization interval \( T_\gamma \), a unique code rate and signal constellation pair is assigned in order to achieve the capacity.

Without loss of generality, the channel capacity can be shown to be a monotonically increasing function of \( \gamma = E_s/N_0 \) subject to a constrained \( R_i \) and \( M_i \) as stated in Equation (12). The required SNR to achieve capacity for a given signal constellation \( M \) is denoted as \( \gamma_{\text{cap}} \). Hence, the instantaneous SNR \( \gamma_s \) that guarantees capacity is given as, \( \gamma_s \geq \gamma_{\text{cap}} \).

Thus the required \( R_b \) bound to achieve capacity in a baseband and spread-spectrum scenarios can be generalized as [12]

\[
1 \geq R \geq \min \left\{ \frac{r}{T \log_2 MB}, \left[ \frac{\gamma_{\text{cap}}}{\gamma_s} \right] \right\} \tag{20}
\]

V. SIMULATION RESULTS

In this section, numerical results are presented to verify the scheme and analyze the performance of the optimization algorithm. The example presented considers an OFDM system based on Turbo coding, the parameters of which are given in Table I. The co-channel interference between sub-carriers is approximated as Gaussian noise, in addition to the AWGN present. The information length of received codewords is thus \( K = 4096 \) bits. The scheme employs Gary-coded M-ary PSK and M-ary QAM modulation with QPSK, 8-PSK, 16-QAM and 64-QAM constellations. The scheme is analyzed for the BER requirements of \( P_\phi = 10^{-2} \) and \( P_\phi = 10^{-4} \) respectively.

The effect of increasing the number of available parameters on the performance of a discrete adaptive scheme is examined in Figures 2(a) and 2(b). The cardinalities of the adaptive parameters used in the adaptive algorithm “2-state”, “4-state” and “6-state” are detailed in Table II. The performance difference between the 2-state and 4-state cardinalities, are negligible. To achieve any appreciable gain in the performance a minimum of 6-state cardinality is required. The 6-state cardinality achieves up to 1 bps/Hz gain in performance over the 2-state cardinality system for the given system configuration over a range, \( J_\phi \).

It is observed that effectively increasing the cardinality during adaptation corresponds to the spectral efficiency approaching ergodic capacity. The effect of \( P_\phi \) on the performance of 2-state, 4-state and 6-state adaptive cardinalities is observed by comparing Figures 2(a) and 2(b). The application of a lower \( P_\phi \) results in a lower spectral efficiency in the adaptive scheme. The effect of constraining \( P_\phi \) results in a lower performance constraint when the available parameters are limited, thus 6-state cardinality responds more favorably to \( P_\phi \) constraint than 2-state cardinality.

The performance of the adaptive scheme is considered across the set of 80 sub-carriers in Figures 3,4,5,6,7 and 8 for varying code rate and signal constellation sets given the BER constraints of \( P_\phi = 10^{-2} \) and \( P_\phi = 10^{-4} \). It is observed that the overall spectral efficiency of the system increases across all sub-carriers as the set of available code rate and signal constellations increase.

In addition, the spectral efficiency decreases with a lowering
(a) Performance of the adaptive Turbo-coded Hybrid-ARQ in OFDM given discrete code rates and signal constellations with $P_\phi = 10^{-2}$

(b) Performance of the adaptive Turbo-coded Hybrid-ARQ in OFDM given discrete code rates and signal constellations with $P_\phi = 10^{-4}$

Fig. 2. Throughput performance of adaptive algorithm in a single carrier of an OFDM system in an AWGN channel
Fig. 3. Throughput performance of 2-state parameter sets of \((R, M)\) employed in adaptation across an 80 carrier OFDM symbol for transmission across a Rayleigh channel of \(\bar{\gamma} = 0dB\) with \(P_\phi = 10^{-2}\).

Fig. 4. Throughput performance of 2-state parameter sets of \((R, M)\) employed in adaptation across an 80 carrier OFDM symbol for transmission across a Rayleigh channel of \(\bar{\gamma} = 0dB\) with \(P_\phi = 10^{-4}\).
Fig. 5. Throughput performance of 4-state parameter sets of \((R, M)\) employed in adaptation across an 80 carrier OFDM symbol for transmission across a Rayleigh channel of \(\gamma = 0\) dB with \(P_{\phi} = 10^{-2}\)

Fig. 6. Throughput performance of 4-state parameter sets of \((R, M)\) employed in adaptation across an 80 carrier OFDM symbol for transmission across a Rayleigh channel of \(\gamma = 0\) dB with \(P_{\phi} = 10^{-4}\)
Fig. 7. Throughput performance of 6-state parameter sets of \((R, M)\) employed in adaptation across an 80 carrier OFDM symbol for transmission across a Rayleigh channel of \(\gamma = 0 dB\) with \(P_\phi = 10^{-2}\)

Fig. 8. Throughput performance of 6-state parameter sets of \((R, M)\) employed in adaptation across an 80 carrier OFDM symbol for transmission across a Rayleigh channel of \(\gamma = 0 dB\) with \(P_\phi = 10^{-4}\)
of the BER constraint applied to the system.

VI. CONCLUSION

In this paper, a throughput maximization algorithm given a required BER constraint was presented for an OFDM system employing Type-III Turbo Hybrid-ARQ at the receiver. The throughput of the system was evaluated for fading channels with varying BER constraints. It is shown that an adaptive approach based on OFDM and Type-III Turbo Hybrid-ARQ achieves near capacity for wideband channels.

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REFERENCES