Abstract—This paper presents a novel technique for computing dangerous voltages due to direct lightning strike into the communication tower and associated earthing system, which is based on the use of the well-known ATP-EMTP software package. The earthing grid and the communication tower structure are approximated by the circular cross section conductors. In numerical model, conductors are subdivided into segments (1D finite elements) and Clark’s model with distributed constant parameters is then applied. Because of the limitations of ATP-EMTP software package, the leakage resistance of buried segments is modelled as additional lumped parameter. Analytical expressions for distributed and lumped segment parameters are derived using the average potential method. Mutual electromagnetic coupling between segments is neglected due to the limitations imposed by the ATP-EMTP software package, which is based on transmission line approach. Separate computer program is developed for obtaining the earth surface transient potential distribution, from which step and touch voltages are then computed.

Index Terms—average potential method, ATP-EMTP, earthing grid, communication tower, lightning transient analysis, step and touch voltages.

I. INTRODUCTION

Knowing the transient behavior of the communication station tower and associated earthing system is very important in the case of direct lightning strikes. A direct lightning strike to the communication tower can cause dangerous overvoltages, which can result in malfunction of the sensitive equipment, as well as dangerous step and touch voltages.

This paper presents a numerical model based on the use of the ATP version of EMTP, which is world-wide, mostly used universal program for digital simulation of electromagnetic transients in power systems [1 - 5]. ATP-EMTP software package is based on transmission line approach, which neglects mutual electromagnetic coupling between segments of conductors.

Elements of the communication station, relevant for the lightning transient study, imply a steel tower structure connected to the earthing grid. Hereafter presented model is thereby divided into two separate parts: model of the communication tower structure, and model of the associated earthing grid. Direct lightning strike to the communication tower is modelled with the Heidler's type of surge current source, [2].

All parameters necessary for the ATP-EMTP simulations are computed by the separate computer program, developed for that purpose. Earth surface transient potential distribution, as well as touch and step voltages are computed from the simulation results obtained by ATP-EMTP using another originally developed computer program.

Advantage of the novel approach for transient step and touch voltages computation is its relative simplicity and short execution time in relation to more complex models based on electromagnetic field theory [6 - 8].

II. MODEL OF THE COMMUNICATION STATION

A. Model of the Communication Tower Structure

Communication tower presents a very complex structure. A simplified model of the tower's structure, which is presented in [9, 10], has been adopted here. Fig. 1 presents an example of 50 m high tower (triangular in cross-section) adopted for the model. Square cross-sectional tower structures can be modeled similarly.

It can be seen from Fig. 1 that the simplified tower structure is, by applying the finite element technique, subdivided into segments (1D finite elements). Each of the tower segments is represented with the distributed constant parameters, according to Clark's model in the EMTP [2]. Input parameters for the Clark's distributed parameters model are:

a) Per-unit-length resistance,
b) Surge impedance,
c) Propagation velocity,
d) Segment length.

Ad a) Per-unit-length resistance of the tower segment can be computed as follows:

\[ R = \frac{\rho_s}{2 \cdot \pi \cdot r_0} \]  

(1)

where:
\( \rho_s \) – resistivity of the tower segment, [\( \Omega \cdot m \)],
\( r_0 \) – equivalent radius of the tower segment, [m].
Surge impedance of the tower segment is defined by the following equation:

\[
Z_s = \sqrt{\frac{L}{C}} = \sqrt{\frac{\varepsilon_0 \cdot \mu_0}{C}}
\]  

where:

- \(L\) – per-unit-length inductance of the tower segment, [H/m],
- \(C\) – per-unit-length capacitance of the tower segment, [F/m],
- \(\varepsilon_0 = 8.854 \cdot 10^{-12} \text{ [As/Vm]}\) – dielectric constant of the vacuum,
- \(\mu_0 = 4\pi \cdot 10^{-7} \text{ [Vs/Am]}\) – relative permeability of the vacuum.

Thus, one needs only to obtain the value of the per-unit-length capacitance for each of the tower segments in order to compute its surge impedance. Per-unit-length capacitance of the tower segment can be computed by means of the average potential method, according to the following expression [10, 11]:

\[
C = \frac{4 \cdot \pi \cdot \varepsilon_0 \cdot \ell}{I_{\text{self}} - I_{\text{mut}}} 
\]  

where \(\ell\) is the length of the tower segment, [m].

According to the average potential method [12 - 14], \(I_{\text{self}}\) and \(I_{\text{mut}}\) are double integrals, which can be defined as follows:

\[
I_{\text{self}} = \int \int \frac{d\ell \cdot d\ell'}{r \Gamma \Gamma'} \]  

where \(r\) is the distance from the source point to the field point, which are located on the different integration curves.

The integration in equation (4) is performed along the segment axis (curve \(\Gamma'\)) and along the curve on segment surface, which is parallel to the segment axis (curve \(\Gamma\)). The first integration in equation (5) is carried out along the axis of segment image (curve \(\Gamma\)), while the second integration is carried out along the curve on segment surface, which is parallel to the segment axis (curve \(\Gamma_s\)). Value of the integral \(I_{\text{mut}}\) depends on the position of the segment relative to the earth surface.

Modal propagation velocity of the surge current through the GSM tower segments is equal to the velocity of light \(c = 3 \cdot 10^8 \text{ [m/s]}\).

Segment length (in meters) should not exceed maximum value which is defined by [9]:

\[
l_{\text{max}} = \frac{c}{6 \cdot f_{\text{max}}} 
\]  

where \(f_{\text{max}}\) is a maximal frequency (in Hz) of interest, found in the lightning surge.

**B. Model of the Earthing Grid**

Each conductor of the earthing grid, by applying the finite element technique, can be subdivided into segments (1D finite elements). A Clark's model with distributed constant parameters is then applied on each segment. Input data for Clark's model are, [2]:

- a) Per-unit-length resistance,
- b) Surge impedance,
- c) Propagation velocity,
- d) Segment length.

Additionally, due to the limitations of the ATP-EMTP software package, the leakage resistance of buried segments is modelled as additional lumped parameter, [2].

**Ad a)** Per-unit-length resistance of the buried conductor segment can be computed according to (1), introducing the resistivity and equivalent radius of the earthing grid conductor.

**Ad b)** Surge impedance of the buried conductor segment is defined by the following equation, [10]:

\[
Z_s = \sqrt{\frac{L}{C}} = \sqrt{\frac{\varepsilon_0 \cdot \varepsilon_r \cdot \mu_0 \cdot \mu_r}{C}} 
\]  

where newly introduced variables have the following meanings:

- \(\varepsilon_r\) – relative dielectric constant of the earth,
\[ \mu_r = 1 \] – relative permeability of the earth.

Hence, in order to compute the surge impedance, one only needs to obtain the value of per-unit-length capacitance of the buried conductor segment. This capacitance can be computed by the average potential method [12 - 14].

Per-unit-length capacitance of the buried conductor segment can be computed according to the following expression [10]:

\[ C = \frac{4 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_r \cdot l}{I_{\text{self}} + I_{\text{mut}}} \]  

where double integrals \( I_{\text{self}} \) and \( I_{\text{mut}} \) are described by expressions (4) and (5).

**Ad c)** Propagation velocity of the lightning surge in the earth can be roughly estimated by the following relation:

\[ v_p = \frac{c}{\sqrt{\varepsilon_r}} \]  

where \( c \) represents the velocity of light, and \( \varepsilon_r \) relative dielectric constant of the earth.

**Ad d)** Each of the buried conductor segments should satisfy the following relation for the maximum length (in meters), [9]:

\[ l_{\text{max}} = \frac{3160}{6} \cdot l^6 \cdot r_0 \]  

where:
- \( \rho \) – resistivity of the earth, [\( \Omega \cdot \text{m} \)],
- \( f_{\text{max}} \) - maximal frequency of interest found in the lightning surge, [Hz].

**Leakage resistance** of the buried conductor segment can be represented in EMTP with the lumped resistance on each side of that segment. Double value of the resistance on each side of the segment is chosen \( 2 \cdot R_L \) in order to obtain value of \( R_L \) after the parallel connection. Value of the buried conductor segment resistance is composed of two terms, as follows, [12, 13]:

\[ R_L = \frac{\rho}{4 \cdot \pi \cdot l^2} (I_{\text{self}} + I_{\text{mut}}) = R_{\text{self}} + R_{\text{mut}} \]  

where:
- \( R_{\text{self}} \) - self resistance of the segment in homogeneous and unbounded medium (earth),
- \( R_{\text{mut}} \) - mutual resistance between segment and its image in relation to the earth surface.

Double integrals \( I_{\text{self}} \) and \( I_{\text{mut}} \) are defined by expressions (4) and (5).

### C. Analytical Solution of Double Integrals

Expressions for computing per-unit-length capacitance of tower structure elements, per-unit-length capacitance of buried conductor segments and leakage resistance of buried conductor segments all involve double integrals \( I_{\text{self}} \) and \( I_{\text{mut}} \). They can be analytically solved, which contributes to the numerical stability of the derived method.

Analytical solution of the double integral \( I_{\text{self}} \) is given by the following expression, [12, 13]:

\[ I_{\text{self}} = 2 \cdot l \cdot \ln \frac{\sqrt{l^2 + r_0^2} + 1}{r_0} - 2 \cdot \sqrt{1^2 + r_0^2} + 2 \cdot r_0\]  

where \( l \) represents segment's length, and \( r_0 \) equivalent radius of the segment.

Analytical solution of integral \( I_{\text{mut}} \) depends on the position of the segment relative to the earth surface. Three different segment arrangements will be examined:
- a) Segment is parallel to the earth surface,
- b) Segment is perpendicular to the earth surface,
- c) Segment is in an asloped position to the earth surface.

**Ad a)** If a horizontal segment is positioned at distance \( h \) [m] parallel to the earth surface, integral solution is [10, 11]:

\[ I_{\text{mut}} = 2 \cdot l \cdot \ln \frac{\sqrt{l^2 + 4 \cdot h^2} + 1}{2 \cdot h} - 2 \cdot \sqrt{1^2 + 4 \cdot h^2} + 4 \cdot h \]  

where:
- \( u_1 = h_1 + h_2 + 1 \)
- \( u_2 = h_1 + h_2 - 1 \)
- \( u_3 = h_1 + h_2 \)

with:

where \( h_1 \) and \( h_2 \) represents distances of the starting and ending points of segment from the earth surface, respectively.
If the segment is in an aslope position relative to the earth surface, analytical solution of the double integral can be written as [10, 11]:

\[
I_{\text{mut}} = 2 \left[ B(x_p, z_p) + B(x_k, z_k) - B(x_p, z_k) - B(x_k, z_p) \right]
\]  

where \( x_p, z_p, x_k \) and \( z_k \) represent coordinates of the starting and ending points of the segment and its image. Terms in (16) are computed using the following relation [10, 11]:

\[
B(x, z) = x \cdot \ln(z - x \cdot \cos \alpha) + \sqrt{x^2 + z^2 + r_0^2 - 2 \cdot x \cdot z \cdot \cos \alpha}
\]

by introducing the following values of coordinate points, and \( \cos \alpha \):

\[
x_p = z_p = \frac{\min \{h_1, h_2\}}{|h_2 - h_1|} \cdot l
\]

\[
x_k = z_k = \frac{\max \{h_1, h_2\}}{|h_2 - h_1|} \cdot l
\]

\[
\cos \alpha = \frac{2 \cdot d^2}{l^2} - 1
\]

where \( d \) represents orthogonal projection of the segment onto the earth surface. Length of the segment is obtained from the following expression:

\[
l = \sqrt{d^2 + (h_2 - h_1)^2}
\]

III. LIGHTNING SURGE MODEL

Lightning surge model used for the simulation is based on the Heidler’s model of current source [2]. Lightning current (as a function of time), in this model is given by the following relation:

\[
i(t) = \frac{I_0}{\eta} \left( 1 + \left( \frac{t}{\tau_1} \right)^n \right)^{-\frac{n}{\eta}} e^{-\frac{t}{\tau}}
\]

where:
- \( I_0 \) – peak value of the lightning current,
- \( \eta \) – correction factor for the peak current,
- \( n \) – factor influencing the rate of rise of the function,
- \( \tau \) – the strike duration; interval between \( t = 0 \) and the point on the tail where the function amplitude has fallen to 50% of its peak value,
- \( \tau_1 \) – duration of the lightning surge front.

IV. EARTH SURFACE POTENTIAL DISTRIBUTION

ATP-EMTP software package without user extensions can only give transient potential distribution on the earthing grid itself. However, more important is an earth surface potential distribution, from which step and touch voltages could be computed. In order to overcome this disability of the ATP-EMTP, a separate computer program has been developed.

From inside the ATP-EMTP, user can request to publish (within results) a leakage currents which each of the earthing grid conductors dissipate into the earth. This is a current flowing through the lumped resistances, which represent the leakage resistance of the buried conductor segment. From this current, an earth surface transient potential distribution can be computed. Knowing the earth surface potential distribution, step and touch voltages can be easily computed.

Fig. 2 shows a single buried conductor segment in a local coordinate system, along with the point \( T(u, v) \) for which the potential needs to be computed.

![Buried conductor segment in the local coordinate system](image)

Potential in the point \( T(u, v) \) at the time instant \( t \), as a consequence of the current \( I \), which segment in the unbounded homogenous medium with resistivity \( \rho \), dissipates into the earth (this current is known from the ATP-EMTP output), can be computed according to the following expression, [12, 13]:

\[
\phi_T(t) = \frac{I}{4 \cdot \pi \cdot l} \int_{-l/2}^{l/2} \frac{du'}{\sqrt{(u-u')^2 + \rho}}
\]
Term expressing the current in the above expression accounts for the potential retardation.

After the analytical solution to the integral in (23), potential in the point \( T(u, v) \) is given by the following expression:

\[
\varphi_T(t) = \frac{\rho}{4\pi} \cdot \frac{I(t - \frac{r_s}{v_p})}{l} \cdot G(u, v) \tag{24}
\]

where \( u \) and \( v \) are local coordinates of the point \( T(u, v) \), \( r_s \) distance in the local coordinate system from the segment's center point to the observation point \( T(u, v) \). Length of the segment is given by \( l \), in meters, while \( v_p \) stands for the velocity of the surge current in the earth, given by (9).

Function \( G(u, v) \) in (24) depends on the geometry of the segment and position of the observation point \( T(u, v) \). It is given by [11 - 13]:

\[
G(u, v) = \ln \left( \sqrt{\left( \frac{u + \frac{1}{2}}{2} \right)^2 + v^2 + u + \frac{1}{2}} \right)
\]

(25)

Local coordinates of the point \( T(u, v) \) can be computed from the global coordinates of the segment starting and ending points and global coordinates of the observation point. Let the starting point of the segment be \( P(x_p, y_p, z_p) \), and ending point of the same segment \( K(x_k, y_k, z_k) \), and let's designate observation point as \( T(x, y, z) \) in the global coordinate system. Origin of the local coordinate system is in the middle point of the segment with global coordinates \( S(x_s, y_s, z_s) \).

Distance between the middle point of the segment and the observation point (expressed with global coordinates) is given by the following expression:

\[
r_s = \sqrt{(x_s - x)^2 + (y_s - y)^2 + (z_s - z)^2} \tag{26}
\]

Local coordinate \( u \) of the observation point can be computed as follows [12]:

\[
u = \frac{2}{l} \cdot [(x - x_s) \cdot (x_k - x_s) + (y - y_s) \cdot (y_k - y_s) + (z - z_s) \cdot (z_k - z_s)] \tag{27}
\]

where the length \( l \) of the segment can be computed as follows:

\[
l = \sqrt{(x_k - x_p)^2 + (y_k - y_p)^2 + (z_k - z_p)^2} \tag{28}
\]

Local coordinate \( v \) of the observation point can be computed with the following expression [12]:

\[
v = \sqrt{(x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2 - u^2} \tag{29}
\]

Potential of the observation point on the earth surface, which is a consequence of leakage currents of \( N \) arbitrarily positioned (mutually connected) earthing grid conductors can be estimated by the following expression:

\[
\varphi_T(t) = \frac{\rho}{2\pi} \cdot \sum_{k=1}^{N} \frac{I_k(t - \frac{r_{sk}}{v_p})}{l_k} \cdot G_k(u, v) \tag{30}
\]

where:

- \( \rho \) – resistivity of the earth, [\( \Omega \cdot m \)],
- \( l_k \) – length of the k-th segment, computed from (28), [m],
- \( G_k(u, v) \) – contribution of the k-th segment to the potential,
- \( I_k \left( t - \frac{r_{sk}}{v_p} \right) \) – leakage current of the k-th segment, [A].

Currents \( I_k \) are obtained from the ATP-EMTP output. The mentioned leakage currents (taking into account the potential retardation) are computed as follows. For each time instant, satisfying the following inequality:

\[
t_m \leq t - \frac{r_{sk}}{v_p} \leq t_{m+1} \tag{31}
\]

cURRENT \( I_k \) of the k-th segment (as a function of time, with potential retardation) is given by the following expression:

\[
I_k \left( t - \frac{r_{sk}}{v_p} \right) = (1 - f_m) \cdot I_k(t_k) + f_m \cdot I_k(t_{k+1}) \tag{32}
\]

where:

\[
f_m = \frac{t - \frac{r_{sk}}{v_p} - t_m}{t_{m+1} - t_m} \tag{33}
\]

Thus, according to (32), it can be seen that the current of the k-th segment is linearly approximated between the two successive time-discret values obtained from ATP-EMTP output. Function \( G_k(u, v) \) depends on the position of the k-th segment in the global coordinate system and can be computed using relation (25).

Using expression (30), transient temporal and spatial
potential distributions on the earth surface can be computed. From those values, step and touch voltages could be easily computed. In order to carry out these computations, separate computer program has been developed.

A. Step and Touch Voltages

By definition, step voltage is a potential difference between two points on the earth surface, which are exactly one meter apart, and can be bypassed by a human step. Similarly, touch voltage is a potential difference between a point on a metallic structure and a point on the earth surface one meter apart from the structure, which can be bypassed by a human touch.

Knowing these potentially dangerous voltages in concrete examples is very important, from the safety aspects. They become available once the transient potential distribution on the earth surface is computed.

B. Simple Earthing Grid Example

In order to demonstrate the above principle for obtaining the earth surface transient potential distribution, a simple earthing grid example is considered. Earthing grid is a square, consisted of four interconnected conductors, each 5 m long, buried at 0.5 m depth. Conductors have a circular cross section, with 1 cm radius. Soil is assumed homogenous with \( \rho = 100 \ \Omega \cdot m \) and \( \varepsilon_r = 8 \).

Fig. 3 shows the geometry involved in the considered simple earthing grid example, along with the position and orientation of the global coordinate system. Observation point on the earth surface with its global coordinates is also given in Fig. 3, as well as a single profile on the earth surface.

Lightning surge current is injected directly into the lower left point of the earthing grid, with the following parameters: amplitude \( I_0 = 1000 \) A, and \( 1/20 \ \mu \text{s} \) shape.

Parameters of the earthing grid conductors were computed according to the presented mathematical model. Simulation of the earthing grid was then carried out in the ATP-EMTP. From the obtained buried conductor segments leakage current distribution, potential on the earth surface is then computed by means of the computer program developed for that purpose.

Temporal transient potential distribution in the point \( T(2.5; 2.5; 0) \) on the earth surface is presented in Fig. 4.

Let us consider a distribution of transient potential along an earth surface profile. Selected profile has a starting point \( T_p(2.5; -5; 0) \) and ending point \( T_k(2.5; 20; 0) \).

Fig. 5 presents the transient potential distribution along this earth surface profile at a time instant \( t = 0.1 \ \mu \text{s} \), Fig. 6 presents the transient potential distribution along the same earth surface profile at time instant \( t = 0.2 \ \mu \text{s} \), while Fig. 7 presents a transient potential distribution along the earth surface profile at time instant \( t = 0.5 \ \mu \text{s} \).

Ordinate axis in Figs. 5 - 7 are in volts, while abscissas are in meters. By comparing the Figs. 5 - 7, wave effects of the transient phenomenon involved could be clearly observed (propagation of the surge).

A separate computer program developed for this purpose allows arbitrary selection of earth surface profiles, as well as individual points. It also allows for the computation of step
and touch voltages.

![Graph showing spatial transient voltage distribution along the earth surface profile at time instant $t = 0.5 \mu s$.](file_Potencial_pravac.adf; x-var x(m); V(x))

**V. COMMUNICATION STATION EXAMPLE**

Complete model of the communication station (model of the tower structure plus a model of the complete earthing grid) is considered in the following example, in order to demonstrate the above presented approach. Treated example is shown in Fig. 8.

![Model of the communication station employed for the numerical computations.](file_Potenocija.png)

Observation point for the computation of transient touch voltage is also shown in the Fig. 8 (point "A"). Touch voltage between this point, located 1m from the tower structure at global coordinates (-1; 0; 0) and tower structure, which could be touched, is computed. Single profile on the earth surface has also been selected in order to compute the transient step voltage distribution (profile "B – B" in the Fig. 8). This profile starts at the following global coordinates: (-1; -20; 0) and extends to the point with global coordinates (-1; 10; 0).

Complete model of the communication station presented in Fig. 8 has been constructed in ATP-EMTP software package. Parameters of tower and earthing grid elements have been previously computed according to the presented mathematical model. Tower elements are approximated with 10 mm radius steel rods, and earthing grid by copper conductors with 6 mm radius. A uniform soil with a 100 $\Omega m$ resistivity, a relative permittivity of 8 and relative permeability of 1 is assumed. Resistance per unit length of the earthing grid segments has been neglected.

Direct lightning strike into the top of the GSM base station's tower has been considered. Parameters of the lightning surge current are $I_0 = 30$ kA and $1/20 \mu s$ shape. Interval of the observation is arbitrarily set to 10 $\mu s$.

Touch voltage between the point "A" shown in the Fig. 8 and tower structure (which could be touched from that point) has been computed. Fig. 9 presents a temporal distribution of the transient touch voltage in that point. Ordinate axis is in volts, while abscissa is in $\mu s$.

![Graph showing transient touch voltage for the observation point (point "A" shown in the Fig. 8).](file_Napon_dodira.adf; x-var t(i); V(t))

Fig. 9. Transient touch voltage for the observation point (point "A" shown in the Fig. 8)

Fig. 10 presents a transient step voltage distribution along the selected profile on the earth surface (profile "B – B" shown in the Fig. 8), for the arbitrarily chosen time instant $t = 5 \mu s$. Ordinate axis in Fig. 10 is in volts, while abscissa is given in meters.

Transient overvoltage in any point on the tower structure (and any other metallic structures connected to it), as well as in any point on the earthing grid can be plotted directly from ATP-EMTP software package.

Apart from that, by means of the developed computer program, one can obtain the transient potential distribution (temporal and spatial) along the earth surface profiles and in the points, which can be arbitrarily chosen. Step and touch voltages are then computed and plotted directly from the same computer program for the selected earth surface points and/or earth surface profiles.
VI. CONCLUSION

Earth surface transient potential distributions, as well as step and touch voltages computations, are performed through extending the well-known ATP-EMTP software package. This extension has been carried out in such a fashion that results from ATP-EMTP present an input data for the developed computer program, which computes earth surface potential distributions (temporal and spatial), as well as transient dangerous voltages.

Due to the limitations inherent to the ATP-EMTP software package, electromagnetic coupling between segments could not be taken into account.

The earth model is limited to the homogenous earth. Soil ionisation effect is not accounted for here.

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