Error Probability Bound Considering Beat Noise in 2-D OCDMA Systems

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Abstract: **Considering a 2-Dimensional Optical Code Division Multiple Access (2-D OCDMA) system using spreading codes in both time and wavelength domains, we study in this paper the impact on the performance of one of the most predominant performance limitation which is beat noise due to the photo detection. The beat noise impact is correlated with the well-known OCDMA limitation named Multiple Access Interference (MAI). Our contribution is to assess, through a theoretical analysis, an error probability bound of a system working in incoherent or partially coherent optical regime. Thanks to the theoretical error probability expression we have developed, the specifications and requirements needed to neglect beat noise effect in a 2-D OCDMA system with a conventional receiver are easily obtained. For a targeted Bit Error Rate (BER), and a given number of active users, one can determine from our results, the 2-D code family parameters, the available data rate and the optical source characteristics, required to be free of beat noise impact.**

Index terms: **beat noise, code division multiple access, multi wavelength optical orthogonal codes, optical networks**

I. Introduction

The Optical Code Division Multiple Access (OCDMA) has received much attention in recent years for next generation of fiber-optic networks [1-4]. Among the numerous OCDMA schemes, incoherent OCDMA systems with coding performed on optical power basis, are considered as the simplest schemes. However, in incoherent OCDMA, code sequences are unipolar (0,1) enforcing Multiple Access Interference (MAI) amount. In addition, compared to 1-Dimensional (1-D) OCDMA systems, 2-Dimensional (2-D) ones with spreading codes in wavelength and time domains [5-7] appear to be of much greater interest. In this paper, we consider an On-Off Keying (OOK) incoherent optical transmission with 2-D OCDMA spreading codes named Multi Wavelength Optical Orthogonal Codes (MWOOC) [6].

At the receiver end, optical signals are detected by a photo detector (PD) performing the square law conversion, giving rise to beat noise [8]. Along with MAI, beat noise strongly degrades the system performance. To quantify the performance degradation due to this limitation, several published works report theoretical studies based on beat noise distribution approximations. In [9-11], beat noise for 1-D systems is assumed to be a Gaussian distributed noise, as in [12] dealing with 2-D systems. Another approximation, named saddle point approximation, is used in [13] to characterize beat noise impact. Moreover, the optical source coherence time is generally taken into account and defines the optical regime coherence, which is independent of the OCDMA system coherence. In previous published works [10], beat noise impact has been estimated for coherent or partially coherent optical regime. It has been neglected for incoherent regime i.e. when the source coherence time is short enough compared to the CDMA chip duration. In the same way, only coherent regime has been treated in [12]. However, even if beat noise and MAI are more important in coherent regime they have also an impact in the incoherent one [9]. To the best of our knowledge, the system specifications in an incoherent regime, free of beat noise have not been assessed yet.

In this paper, we consider an incoherent or partially coherent optical regime. The main contribution of our work is to evaluate, with a simple model, the beat noise impact on the 2-D OCDMA system performance considering the worst interference pattern scenario. We develop in this case a theoretical analysis of errors linked to beat noise for an incoherent OCDMA system using MWOOC codes.

From a parametric study, the system parameters (code parameters and optical source coherence time) leading to a given performance with a conventional receiver can be assessed. Our results complement the ones given by literature as they are theoretically given for an incoherent and partially coherent regime where beat noise is usually neglected.

This paper is organized as follows. In section II, we present the 2-D OCDMA system. The analysis of beat noise effect is developed in section III. The error probability formula taking into account the beat noise is established in section IV. A 2-D code parametric study is done in section V. Finally, we conclude on the system parameters needed to implement an OCDMA system and to neglect beat noise impact.

II. System Description

A. 2-D Multi Wavelength Optical Orthogonal Codes (MWOOC)

2-D OCDMA coding has recently attracted much attention and is the most promising technique for practical OCDMA

Manuscript received October 22, 2007 and revised February 26, 2008.

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applications. We consider a 2-D time-wavelength spreading construction based on code matrices of dimension ($L\times F$) named MWOOC. L and F respectively refer to spectral and temporal spreading lengths. These codes, described in [6], are derived from the well known 1-D temporal codes named Optical Orthogonal Codes (OOC) [1]. For a MWOOC noted ($L\times F$, W), F is a prime number, and the number of wavelengths L is equal to the code weight W. The temporal length F defines the chip time $T_c = T_b/F$, where T_b is the bit time. Because we consider unipolar codes for the OOK transmission, the auto-correlation (degree of similarity between the code and itself, for non zero shift) and the cross-correlation (degree of similarity between two distinct codes) cannot be null, and we consider the minimal value of 1. The 2-D encoding is performed such as only one chip is set to '1' per wavelength and the code cardinality is given by:

$$N_{\max} = L + F \qquad (1)$$

As the code sequences are not strictly orthogonal, the Multiple Access Interference (MAI) constitutes a performance limitation. An overlap between two codes occurs only when both wavelength and time chips coincide from two code matrices and when a '1' datum is emitted from the interferer user. Since there is up to one chip set to '1' per row in a 2-D code matrix, the probability that two matrices have a '1' at the same wavelength is W^2/L. Furthermore, considering two '1', at the same wavelength and from different code matrices, the probability that they are on the same time slot is $1/F$. Therefore, the interference probability is:

$$p = p_{2D} = \frac{W^2}{2L\cdot F} = \frac{L}{2\cdot F} \qquad (2)$$

This means that for a 2-D MWOOC ($L\times F$, $W=L$), the MAI has the same distribution as for classical 1-D OOC (F', W) considering an equivalent temporal length $F'=L\times F$ with a weight $W=L$.

B. Transmission link

We consider that each user emits independent equiprobable binary data. The 2-D OCDMA encoder (Fig.1) performs the multiplication of the emitted data by the corresponding user code matrix. Then, coded data from different users are summed and emitted at different wavelengths. We do not focus here on the practical implementation of the encoder, which can be an optical or a partially optical system.

At the receiver end, we consider the simplest receiver named Conventional Correlation Receiver (CCR), performed in electric domain. The different wavelengths are first separated by a demultiplexer. The optical signal is then converted into an electrical signal on each wavelength by the photo detector (PD) (Fig.1). After the opto-electrical conversion, the signal is correlated with the corresponding desired user matrix row. Undesired users, whose code has an overlap with the desired user one, can induce errors at the data recovery. Indeed, undesired chips are added to the desired user ones and enforce the decision variable value. This constitutes the MAI, and is inherent to the OCDMA technique. Therefore, in a noiseless case, for a '0' datum emitted, if the decision variable is higher than the threshold level S of the decision device, an error occurs. However, for $0 < S \le W$, an error cannot occur on a '1' emitted datum since the decision variable value is never lower than W. An upper bound of the error probability for N active users has been obtained [1], in a chip synchronous case for $0 < S \le W$:

$$P_{CCR} = \frac{1}{2}\cdot\sum_{i=S}^{N-1}\binom{N-1}{i}(p)^i(1-p)^{N-1-i} \quad . \qquad (3)$$

where p is the interference probability given by (2).

In the following, we have considered that S is set to its optimal value, corresponding to a minimal error probability: S=W.

C. Practical limitations

To limit the MAI and allow a high number of active users for a given performance, the temporal spreading length and number of wavelengths values have to be high (from (3)). However, practical limitations impose some constraints on the parameters.

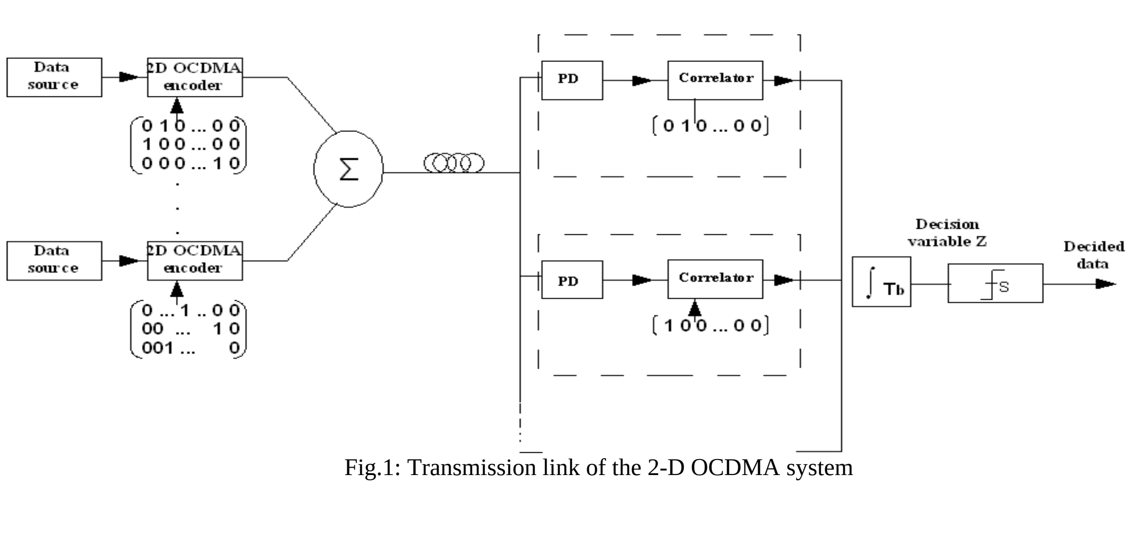

Fig.1: Transmission link of the 2-D OCDMA system

First, the number of wavelengths cannot be too high. Actually, if there are more wavelengths than active users, the interest of using OCDMA with 2-D codes instead of Wavelength Division Multiple Access (WDMA) is poor. Moreover, to emit a signal which is spread on L wavelengths, one can use either a multi-wavelength source, or L distinct sources. The first solution, multi-wavelength source, is all the more hard to produce as the number of wavelengths is high. The other way consisting in using L distinct sources constitutes a heavy system when L becomes too high. Consequently, the number of wavelengths has to be as low as possible. For the considered codes (MWOOC) the number of wavelengths is equal to the minimal value it can take that is the code weight.

The temporal spreading value F is another important limitation. In order to lower the Bit Error Rate (BER), it should be set to a very high value. However, it is linked to the data rate D and the photodetector bandwidth B. The chip rate $D_c=1/T_c=FD$ is limited by the bandwidth following:

$$D_c=FD<B\ . \tag{4}$$

Consequently, for a given bandwidth, when F increases, the available data rate D is strongly limited. Therefore, in order to satisfy high speed communication data rates, F cannot be set too high.

Finally, the optical source is characterized by the minimal pulse time it can deliver. Thanks to the available technology, pulses can be very short, but when dimensioning the system, it is interesting to choose a pulse time value not too small to be less restrictive for practical realization.

In the following, we investigate the impact of beat noise, which is along with the MAI, an important performance limitation due to optical and opto-electrical devices. Considering an incoherent optical regime and a partially coherent one, we take into account all the limitations to dimension the system.

III. BEAT NOISE ANALYSIS

In this analysis, we use a large number of symbols, which are listed in Table I.

TABLE I

SYMBOL DEFINITION

Symbols	Signification	
F	Temporal code length	
W	Code weight	
L	Number of wavelengths	$L=W$ for MWOOC
S	Threshold level of the CCR decision device	$S=W$
T_b	Bit time	
T_c	Chip time	$Tc=Tb/F$
N_{max}	Code cardinality	$N_{\max}=L+F$
N	Number of active users	$N<N_{max}$
$E(t)$	Incident optical field	$I_{ph}=K.\int_{T_c}\lvert E(t)\rvert^2 dt$
K	Photodiode responsivity	
I_{ph}	Photocurrent	
A_d	Desired user data over the chip time	
A_i	Undesired #i user data over the chip time	
B	Photodetector bandwidth	
T_p	Pulse time	
P	Number of pulse time in the chip time	$P=T_c/T_p$
m	Number of undesired user chips over the chip duration	
m_n	Number of interfering pulses on the n^{th} position in the chip time ($1\le n\le P$)	
d	Emitted data	0 or 1
$\hat{d}$	Decided detected data	
Z	Decision variable	
i	MAI term = number of interfering pulses in the bit time before photodetection	
k	Beat noise term	
I	Global term of interference (MAI+beat noise)	$I=i+k$
Q	Number of occupied pulse time slots in the bit time	$0\le Q\le P\cdot W$
a_i	Pulse amplitude of the i^{th} pulse time slot	

According to several studies, beat noise, appearing at the photo detection when several incident optical fields are interfering, is a major limitation for OCDMA systems [8-13]. At the receiver end, an incident optical field $E(t)$ is converted into a photocurrent I_{ph} over the chip duration T_c, following the square law detection:

$$I_{ph}=K.\int_{T_c}\left|E(t)\right|^2 dt\ . \tag{5}$$

K is the photodiode responsivity (A/W) and depends on the photodetector.

Assuming that there are m undesired user chips over the chip duration, the incident field can be expressed as:

$$E(t)=A_d\cdot e^{j(\omega_d t+\phi_d(t))}+\sum_{i=1}^{m}A_i\cdot e^{j(\omega_i(t-\tau_i)+\phi_i(t-\tau_i))}\ . \tag{6}$$

A_d and A_i respectively correspond to the desired user data field amplitude and to the undesired user data one, interfering over the chip time. ω_d and ω_i are their respective pulsations, ϕ_d and ϕ_i are the relative phase noise of each signal, and τ_i is the relative network transit delay of the i^{th} interfering pulse compared to the desired one.

In the following, we assume that each emitted light pulse is represented by a rectangular pulse of duration T_p (Fig.2). The pulse position is randomly chosen over the chip duration T_c. Depending on the ratio T_c/T_p, the working regime is named coherent ($T_p\ge T_c$), partially coherent ($T_p\le T_c$) or incoherent ($T_p<<T_c$), (Fig.2) [10]. In this study, we consider an incoherent regime, or a partially coherent regime. The minimal T_p value corresponds to the shortest pulse time the source can produce. Consequently, the T_p value depends on the source coherence time τ [9], which constitutes its lower bound.

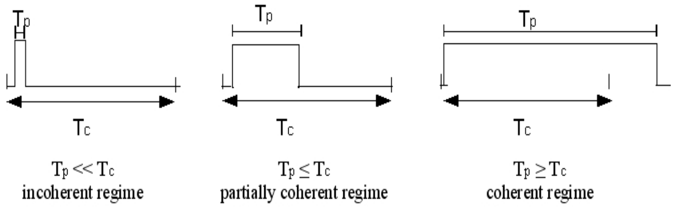

Fig.2: Pulse time T_p versus chip time T_c, defining the regime coherence

Moreover, we assume that the difference between two different frequencies is small enough compared to the chip rate: $(\omega_i - \omega_j)/2\pi << 1/T_c$ [10]. When calculating the current over a chip time with (5) and (6), the cross terms $(\omega_i - \omega_j)\cdot t$ can thus be neglected in the integral. Therefore, after the photodetection process, the current is expressed over a chip time as:

$$I_{ph} = K\cdot\int_{T_c}\left|E(t)\right|^2 dt = KT_pA_d^2 + KT_p\sum_{i=1}^{m}A_i^2 +$$

$$2K\sum_{i=1}^{m}A_dA_i\int_{T_c}\mathbf{cos}(\Delta\phi_{di}-\omega_i\tau_i)\,dt + 2K\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}A_iA_j\int_{T_c}\mathbf{cos}(\Delta\phi_{ij})\cdot dt \quad (7)$$

where $\Delta\phi_{xy} = \phi_x(t-\tau_x) - \phi_y(t-\tau_y)$.

The first term of (7) corresponds to the desired user data and the second to the MAI contribution from undesired users. The third term constitutes the signal-interferer beat noise and the last term the interferer-interferer beat noise. Note that beat noise terms appear only if MAI is present over the chip ($A_i \neq 0$).

From now on, we consider that the ratio T_c/T_p is an integer value P, and that pulses can be either on the same T_p interval or on distinct intervals: we do not consider the case where they are partially overlapping. On each position, there is thus a positive or null integer number of interfering pulses. Therefore, we can decompose the terms of equation (7) as a sum over the P intervals of the chip time, with m_n the number of interfering pulses on the n^{th} pulse position:

$$I_{ph} = KT_pA_d^2 + KT_p\sum_{i=1}^{m}A_i^2 + 2K\sum_{n=1}^{P}\sum_{i=1}^{m_n}A_dA_i\int_{T_p}\mathbf{cos}(\Delta\phi_{di}-\omega\tau_i)\,dt$$

$$+2K\sum_{n=1}^{P}\sum_{i=1}^{m_n-1}\sum_{j=i+1}^{m_n}A_iA_j\int_{T_p}\mathbf{cos}(\Delta\phi_{ij})\cdot dt\,. \quad (8)$$

As we suppose that each user occupies a single pulse position over the chip time, A_d is only non null over one position (from 1 to P), named the d^{th} position. The summation in the third term of (8) can thus be reduced to only one term for $n=d$:

$$I_{ph} = KT_p(A_d^2 + \sum_{i=1}^{m}A_i^2) + 2K\cdot A_d\sum_{i=1}^{m_d}A_i\int_{T_p}\mathbf{cos}(\Delta\phi_{di}-\omega\tau_i)\,dt$$

$$+2K\sum_{n=1}^{P}\sum_{i=1}^{m_n-1}\sum_{j=i+1}^{m_n}A_iA_j\int_{Tp}\mathbf{cos}(\Delta\phi_{ij})\cdot dt \quad (9)$$

Moreover, we assume the same considerations as in [10], i.e. $\omega\tau_i << 2\pi$ is nearly constant over the chip duration and $\Delta\phi_{ij}$ constant over each pulse time. In this case, $\omega\tau_i$ is time independent over T_c, so the first integral $\int_{T_p}\mathbf{cos}(\Delta\phi_{di}-\omega\tau_i)\,dt$ is reduced to $\int_{T_p}\mathbf{cos}(\Delta\phi_{di})\,dt$. Then, as $\Delta\phi_{xy}$ is constant over T_p, the integral can be suppressed. Therefore, (9) is simplified to:

$$I_{ph} = KT_p(A_d^2 + \sum_{i=1}^{m}A_i^2) + 2KT_p\sum_{i=1}^{m_d}A_dA_i\,\mathbf{cos}(\Delta\phi_{di})$$

$$+2KT_p\sum_{n=1}^{P}\sum_{i=1}^{m_n-1}\sum_{j=i+1}^{m_n}A_iA_j\,\mathbf{cos}(\Delta\phi_{ij}) \quad (10)$$

We now evaluate the impact on the photocurrent value I_{ph} of the phase noise $\mathbf{cos}(\Delta\phi_{xy})$, representing the difference of delay between two distinct fields, when a ‘1’ or a ‘0’ datum is emitted by the desired user. To assess the possible values taken by I_{ph}, each variable of (10) is listed.

Normalizing each emitted datum amplitude by 1, we can determine the probabilities that A_i are equal to ‘1’ or to ‘0’ over the chip time (first sum of (10)):

$$\begin{cases} p_{chip}(A_i=1) = p(d=1)\cdot\dfrac{1}{FL} \\ p_{chip}(A_i=0) = p(d=0) + p(d=1)\cdot\left(1-\dfrac{1}{FL}\right) \end{cases}. \quad (11)$$

Assuming an equiprobable emission, we obtain:

$$\begin{cases} p_{chip}(A_i=1) = \dfrac{1}{2FL} \\ p_{chip}(A_i=0) = 1-\dfrac{1}{2FL} \end{cases}. \quad (12)$$

On the same manner, for the second and third sums of (10), where A_i and A_j values are defined over the pulse time, the probabilities that A_i are equal to ‘1’ or to ‘0’ in the pulse time are given by:

$$\begin{cases} p_{pulse}(A_i=1) = \dfrac{1}{2FLP} \\ p_{pulse}(A_i=0) = 1-\dfrac{1}{2FLP} \end{cases}. \quad (13)$$

The remaining random variable in (10) is $\mathbf{cos}(\Delta\phi_{xy})$, which is supposed to be uniformly distributed between -1 and 1 in a first time.

By simulating all these random variables, each following their own distribution law, we can assess the probabilities p_0 and p_1 defined by: $p_0 = p(I_{ph}/KT_p > 1/2 \mid d=0)$ and $p_1 = p(I_{ph}/KT_p < 1/2 \mid d=1)$, with d the emitted desired user datum. Results have been reported in Table II for a given MWOOC and with N=30 active users, for several P values with P>1. We can remark that in all cases $p_1 < p_0$. Moreover, when P increases, p_1 becomes negligible compared to p_0. This means that the MAI positive contribution compensates in most cases the negative beat noise contribution.

TABLE II

PROBABILITIES P_1 AND P_0 OBTAINED WITH A MWOOC(59X7) WITH N=30 ACTIVE USERS FOR SEVERAL P VALUES.

P	$p_1 = p(I_{ph}/KT_p < 1/2 \mid d=1)$	$p_0 = p(I_{ph}/KT_p > 1/2 \mid d=0)$
2	6.5×10^{-3}	3.4×10^{-2}
5	2.6×10^{-3}	3.4×10^{-2}
10	1×10^{-3}	3.7×10^{-2}
20	9×10^{-4}	3.8×10^{-2}
30	6×10^{-4}	4.1×10^{-2}

Consequently, errors made on ‘1’ are much less frequent than the one made on ‘0’. This result is in accordance with the one published in [10]. Actually, the global noise probability density function (corresponding to MAI, beat noise and thermal noise) presented for $P>1$ ($kt>1$ with their notations) is not centered, and the negative values are all the more negligible as P increases. The global noise contribution is thus rarely negative, and the negative beat noise contribution on ‘1’ is mainly compensated.

Therefore, in the following, we only focus on errors made on ‘0’, to develop a theoretical bound of the error probability. We consider the worst case of interference for the ‘0’ which is given when all contributions in (10) are positive and maximal. We do not thus consider anymore that $\cos(\Delta\phi_{xy})$ is uniformly distributed, but that it is always equal to its maximal positive value of 1. This worst case ($\cos(\Delta\phi_{xy})=1$), means that the square law detection applied on each slot of duration T_p is always entirely constructive.
We develop in the next part an analysis of pulse repartition in order to obtain a theoretical error probability bound.

IV. Error Probability In A Beat Noise Corrupted Case

A. Context

Let d be the emitted datum and $\hat{d}$ the detected datum. The error probability is given by:

$$P_e = \tfrac{1}{2}\cdot prob(\hat{d}=0 | d=1) + \tfrac{1}{2}\cdot prob(\hat{d}=1 | d=0)\,. \quad (14)$$

We note Z the decision variable corresponding to the datum d. This value is compared to the threshold value S, ($0<S\le W$) and following the decision rule, the error probability can be expressed as:

$$P_e = \tfrac{1}{2}\cdot prob(Z<S | d=1) + \tfrac{1}{2}\cdot prob(Z\ge S | d=0)\,. \quad (15)$$

Z can be developed as: $Z = W\cdot d + I$, where I is a global term of interference in the bit time (MAI+beat noise). Therefore, the error probability becomes:

$$P_e = \tfrac{1}{2}\cdot prob(W+I<S) + \tfrac{1}{2}\cdot prob(I\ge S)\,. \quad (16)$$

From now on, we consider the worst case of beat noise impact ($\cos(\Delta\phi_{xy})=1$) with $S=W$. Consequently, the contribution of global noise is a positive value, $I\ge 0$ and the first term of (16) is null.

To calculate the remaining second term, we define all interference cases that lead a ‘0’ to be decided as a ‘1’. These cases can be divided into two categories: a classical case where MAI is high enough to generate errors, even without considering the beat noise, and a case where errors are due to MAI+beat noise.

To distinguish these two categories, we express the term of interference as: $I=i+k$, with i the MAI term contribution and k the beat noise term contribution in the bit time:

$$prob(I\ge S) = prob(i+k\ge S \cap i\ge S) + prob(i+k\ge S\cap i<S)$$
$$= prob(i\ge S) + prob(i<S)\cdot prob(i+k\ge S | i<S)\,. \quad (17)$$

The first term of (17) corresponds to the case where errors appear even without beat noise. It is given by [1]:

$$prob(i\ge S) = \sum_{j\ge S} prob(i=j) = \sum_{i=S}^{N-1}\binom{N-1}{i}(p)^i(1-p)^{N-1-i}\,. \quad (18)$$

with p the interference probability given by (2) for 2-D codes. The second term evaluates the effect of errors due to beat noise and MAI altogether:

$$prob(i<S)\cdot prob(i+k\ge S | i<S) = \sum_{j=0}^{S-1} prob(i=j)\cdot prob(j+k\ge S)$$
$$= \sum_{i=0}^{S-1}\binom{N-1}{i}(p)^i(1-p)^{N-1-i}\cdot prob(i+k\ge S)\,. \quad (19)$$

The unknown term in (19) is: $prob(i+k\ge S)$, for a i value fixed between 0 and $S-1$ (i corresponding to the number of interfering pulses in the bit time). To obtain this probability, we first list all possible repartition cases of the i interfering pulses in the bit time without considering beat noise term k, before evaluating which ones give rise to an error.

B. Repartition of interfering pulses

As previously, we consider that the chip time T_c is divided into P slot times of duration T_p: $T_c = P\cdot T_p$, where P is an integer. Each interfering pulse is assumed to occupy a pulse time T_p. With this assumption, there are P available slots for a light pulse in a chip time T_c. Besides, according to the 2-D coding rule, there are W chips set to ‘1’ in the bit time. Thus, the interfering pulses are in the W chips of the desired user code, and consequently there are $P\cdot W$ positions of interest in the bit time, where the interfering pulses can be distributed.

Let i be the number of interfering pulses before photodetection (only MAI contribution). They can occupy from 1 to i slots. For a fixed number of occupied slots Q, there are $\binom{P\cdot W}{Q}$ possibilities to select the Q positions among $P\cdot W$ slots.

We note a_m, with $m=1...Q$, the non null positive amplitude value of each occupied slot. We thus have $\sum_{m=1}^{Q} a_m = i$.

On the first occupied slot, the pulse amplitude is a_1 and can take values from 1 to $i_1=i$. Thus, there are $\binom{i_1}{a_1}$ possibilities to select a_1 pulses among i. Then, if $a_1\ne i$, we consider the second occupied slot whose amplitude a_2 can now take values from 1 to $i_2=i-a_1$. There are $\binom{i_2}{a_2}$ possibilities to choose a_2 interfering pulses among i_2. In a general manner, for the m^{th} occupied slot, $i_m = i-\sum_{q=1}^{m-1} a_q$ and there are $\binom{i_{2m}}{a_m}$ possibilities to choose a_m interfering pulses among i_m.

By following this enumeration until the Q^{th} slot, we can deduce for each fixed Q-uplet ($a_1,a_2..,a_Q$), the number of repartitions of the i interfering pulses:

$$\binom{i_1}{a_1}\binom{i_2}{a_2}\cdots\binom{i_m}{a_m}\cdots\binom{i_{Q-1}}{a_{Q-1}}\cdot\binom{P.W}{Q}\,. \quad (20)$$

with $a_1 \in [1;i_1], a_2 \in [1;i_2], \ldots a_m \in [1;i_m], \ldots a_{Q-1} \in [1;i_{Q-1}], a_Q = i - \sum_{q=1}^{Q-1} a_q$.

C. Error Probability

Once all MAI repartition cases have been numbered, only the ones conducing to an error after the square law detection, must be taken into account in the error probability expression. After the photo detection process, the interference term I is composed of MAI ($i= \sum_{m=1}^{Q} a_m$) and beat noise (k).

It is obtained by applying the square law detection on each slot:

$$I = \sum_{m=1}^{Q} a_m^2 = i + k \; . \tag{21}$$

There is finally an error if the total interference I given by (21) is higher than the threshold level S. Thus, the probability that a case gives rise to an error can be noted as:

$$p_s^{(Q)} = prob^{(Q)}(I \geq S) = \begin{cases} 1 \;\; if \;\; \sum_{m=1}^{Q} a_m^2 \geq S \\ 0 \;\; if \;\; \sum_{m=1}^{Q} a_m^2 < S \end{cases} . \tag{22}$$

As there are $P{\cdot}W$ positions of interest for the pulses, the probability that the i interfering pulses are distributed over the $P{\cdot}W$ positions is: $\left(1/(P \cdot W)\right)^i$. So, when multiplying each case enumerated in (20) by the corresponding probability, we obtain after summing for each possible Q-uplet ($a_1,a_2..,a_Q$) value, the probability searched in (19):

$$prob(i + k \geq S | i < S) = \sum_{a_1=1}^{i_1} \sum_{a_2=1}^{i_2} \ldots \sum_{a_m=1}^{i_m} \ldots \sum_{a_{Q-1}=1}^{i_{Q-1}}$$

$$\binom{i_1}{a_1}\binom{i_2}{a_2}\ldots\binom{i_m}{a_m}\ldots\binom{i_{Q-1}}{a_{Q-1}} \cdot \binom{P.W}{Q} p_s^{(Q)} \cdot \left(\frac{1}{P.W}\right)^i . \tag{23}$$

By replacing (23) in (19), and then in (17) we finally obtain the error probability expression:

$$P_e = \frac{1}{2} \cdot \sum_{i=S}^{N-1} \binom{N-1}{i} (p)^i (1-p)^{N-1-i}$$

$$+ \frac{1}{2} \cdot \sum_{i=0}^{S-1} \binom{N-1}{i} (p)^i (1-p)^{N-1-i} \sum_{a_1=1}^{i_1} \sum_{a_2=1}^{i_2} \ldots \sum_{a_m=1}^{i_m} \ldots$$

$$\sum_{a_{Q-1}=1}^{i_{Q-1}} \binom{i_1}{a_1}\binom{i_2}{a_2}\ldots\binom{i_m}{a_m}\ldots\binom{i_{Q-1}}{a_{Q-1}} \cdot \binom{P.W}{Q} \cdot p_s^{(Q)} \cdot \left(\frac{1}{P.W}\right)^i . \tag{24}$$

with $i_m = i - \sum_{q=1}^{m-1} a_q$.

D. Validation of theoretical error probability expression

To validate our theoretical expression (24), we have compared our results to published ones [9]. In order to be in the same configuration than in [9], we have applied our theoretical expression to 1-D Optical Orthogonal Codes [1], with $p = \frac{W^2}{2F}$.

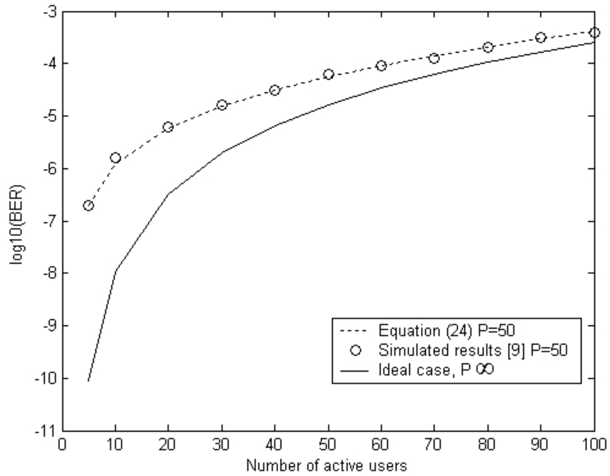

Fig.3: Comparison of the BER for the OOC (2197,4) in presence of beat noise (P=50) from our analysis and from Gaussian approximation [9]

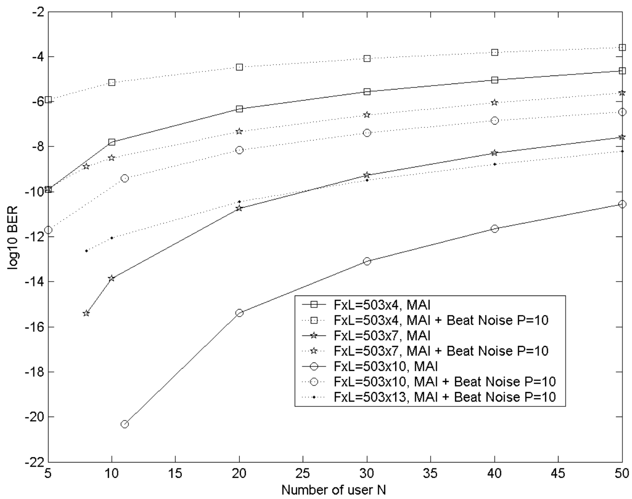

Fig.4: BER for MWOOCs (LxF,L=W), F=503, W=L=4,7,10,13 with MAI and MAI+beat noise (P=10)

The theoretical error probability of the OOC ($F=2197$, $W=4$) with $P=T_c/T_p$=50 and for several numbers of user values has been plotted in Fig.3. We have also reported the values obtained by simulation of beat noise Gaussian distribution from the work presented in [9] for the same OOC parameters and P value.

It can be pointed out that the curve obtained from our theoretical formula fits the one obtained in [9] by simulation. Thus, considering the worst scenario of a constant phase noise, gives a good valuation of the error probability obtained assuming that beat noise has an uniform distribution. The main interest of our work is that low BER values can be theoretically obtained whereas they cannot be reached by numerical simulations.

For example, we have plotted in Fig.4 the theoretical BER in the ideal case with only MAI (solid lines) and in a beat noise case with P=10 (doted lines), for several 2-D code families. In all cases, the temporal length F has the same value but the number of wavelengths varies. We can first remark that beat noise has not the same impact in all the cases. For low BER values, beat noise has a more significant impact on the performance than for high BER values which corresponds to a high MAI amount. We also deduce from these curves that, in order to reach a given specification, for example BER=5.10^{-10} for N=30 users with F=503, the number of wavelengths required has to be much higher when beat noise is taken into account. Actually, in the ideal case, this specification is reached with L=7 wavelengths, whereas, with beat noise, L=13 wavelenghts are required. Therefore in this case, beat noise has an important impact on the system parameters. This is an expected result since the studied case (P=10) is close to the coherent regime (P=1), corresponding to a case where the beat noise is a very important limitation.

In order to evaluate beat noise impact as a function of the optical regime coherence, we have reported in Fig.5 the BER as a function of $P=T_c/T_p$ for the MWOOC (FxL=59x7, W=L=7) with N=32 users.

We remark first that beat noise impact on performance reduces as P increases. Indeed, the probability that two different pulses of two users interfere, decreases with the number of positions in the chip interval. Therefore, for high P values i.e. in the incoherent regime, ($T_p << T_c = P \cdot T_p$), the performance converges, as expected, to the ideal one (taking into account only MAI). This result gives a complementary validation of the theoretical error probability expression (24).

Finally, to reach low BER, two parameters are important: first the temporal code length or the number of wavelengths has to be high, then the P value has to be high. However, a high temporal length implies a low data rate value and a high P value corresponds to small pulse time T_p. Consequently, there is a tradeoff between the OCDMA system parameters and the source characteristics. In the next part, we study this tradeoff by developing a parametric study.

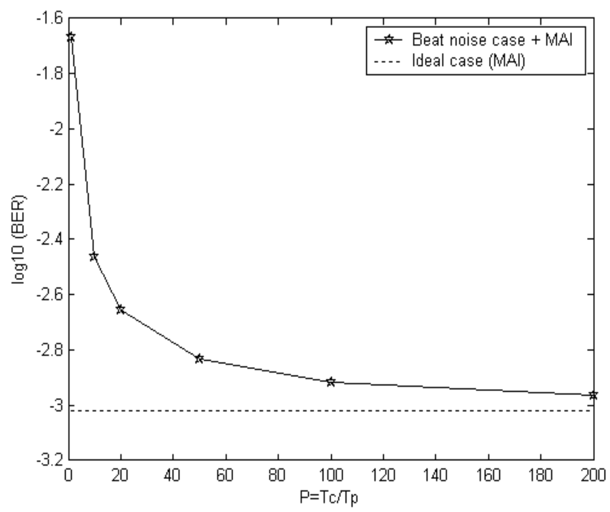

Fig.5: BER for MWOOC (59x7,7) with N=32 users versus the ratio $P=T_c/T_p$

V. Parametric study and Performance analysis

From the error probability expression (24) and for a given MWOOC family and number of users N, we can determine the conditions on the system parameters to neglect the impact of beat noise on the performance. We particularly extract the P_{min} value necessary to be free of beat noise impact. This P_{min} value is linked to the pulse time T_p value for a given data rate D as follows:

$$T_p = 1/(P \cdot F \cdot D)\,. \quad (25)$$

Moreover, the bandwidth B limits the chip rate D_c and the bit rate D (4). In order to provide the highest available data rate for a given code and a given bandwidth, we choose the limit condition:

$$F \cdot D = B\,. \quad (26)$$

Therefore, for a given code (FxL,W=L) and its corresponding P_{min} value needed to be free of beat noise, the maximal T_p value is obtained from:

$$T_p = 1/(P_{\min} \cdot B)\,. \quad (27)$$

We first determine from (3) and for several W values, the 2-D code parameters corresponding to a minimal temporal length, i.e. a maximal data rate, for a given B value. The specified performance is a BER<10^{-9} for N=32 users in a noiseless case. Considering beat noise in addition to MAI, we then evaluate for each code by using the analytical expression (24), the P_{min} value leading to the same performance as in the ideal case (only MAI). Finally, we calculate the corresponding maximal T_p value (27). Results are reported in Table III.

We can note that when F and W increase, the P_{min} value decreases. Consequently, for a fixed bandwidth it makes T_p increase. A high T_p value is preferred, since it is less restrictive for practical optical source. The MWOOC family choice also depends on the corresponding available data rate which it searched to be as high as possible.

TABLE III

Minimal P value needed to be free of beat noise for minimal 2-D codes respecting BER<10^{-9} with 32 users with a CCR. The corresponding T_P values and available data rates are also presented.

$L \times F, W$	P_{min}	$D_{max}=B/F$ (Mbit/s)	$T_{p\,max}$ (ps) $=1/(P_{min}B)$
10×211,10	3393	47.39	0.03
11×173,11	2989	57.80	0.03
12×149,12	1387	67.11	0.07
13×157,13	206	63.69	0.49
14×191,14	50	52.36	2.00
15×211,15	25	47.39	4.00
16×241,16	18	41.49	5.56
17×277,17	12	36.10	8.33
18×307,18	8	32.57	12.50
19×347,19	4	28.82	25.00
20×383,20	3	26.11	33.33
21×421,21	2	23.75	50.00
22×463,22	2	21.60	50.00
23×509,23	2	19.65	50.00
24×557,24	1	17.95	100.00
25×601,25	1	16.64	100.00

However, the maximal available data rate (67 *Mbit/s*), obtained for the MWOOC (12×149,12)) corresponds to a short pulse time around 0.07 *ps*. Conversely, the maximal pulse time (T_p=100*ps*) corresponds to a low data rate of 18 *Mbit/s*.

In order to explore more precisely this tradeoff, we have considered 2-D codes with different W values. The W values have been chosen just upper the W value of the minimal length code in table III, i.e. the MWOOC (12×149,12). For W values of 13,14 and 15, we have thus searched the minimal P value to be free of beat noise from (24), by varying the temporal code length F. It is possible to represent the variations of $D=B/F$ as a function of $T_p = 1/(P_{\min} \cdot B)$ for a given B. The results are reported in Fig.6 with B=10 GHz.

The same study has been done for 2-D codes with a BER<10^{-5}. In this case, the weights have been fixed to W=9, 10, 11, and 12 because with 32 users and a BER<10^{-5}, the minimal MWOOC is (9×89,9). The couples (D, T_p) have been plotted in the same figure as for BER<10^{-9} (Fig.6).

As expected, the data rates D obtained with a BER<10^{-5} (up to 115 *Mbit/s*) are higher than those obtained with a BER<10^{-9} (up to 67 *Mbit/s*). Actually, the minimal code lengths F are lower as the BER is higher, therefore the data rate D increases when the BER increases.

It can also been remarked that for each weight value W, the available data rate is defined on distinct range of values. For example, for a BER<10^{-9} and a weight W=12, the data rate is up to 67 *Mbit/s* whereas it is up to 50 *Mbit/s* with W=15. Therefore, for a given weight, the data rate is bounded by a maximal value, corresponding to the minimal code length value F.

Moreover, as we have seen from Fig.4, for high BER values, the beat noise impact is not preponderant compared to the MAI effect. For a given weight W, several codes of different lengths have thus the same minimal P value, to be free of beat noise impact. This explains that there are several T_p value floors in Fig.6 for BER<10^{-5}.

From the results of Fig.6, it can be pointed out that, whatever the targeted BER is, for a given weight W, i.e. for a given number of wavelengths (L=W), when the targeted data rate value increases, the corresponding pulse time decreases. However, a short pulse time is more restrictive for optical source requirements.

In addition, for a targeted data rate value, when the weight W increases, the pulse time value is higher, which is more suitable for optical source characteristics. However, the number of wavelengths increase introduces more system complexity and more conditions on the optical source bandwidth and on the maximal available data rate.

To sum up, from our theoretical results, one can determine the conditions over the system parameters (D, T_p, W) required to be free of beat noise for a given performance. If a data rate of 100Mbit/s is required, (Fig.6) we can note that it is achievable for a BER<10^{-5} with two different couples: (T_p=1.5 ps, W=9) or (T_p=3.5 ps, W=10). However, this data rate is not achievable for a BER<10^{-9}. As another example, a targeted data rate of 50Mbits/s can be reached with W=12, 13, 14 for a BER<10^{-9} and with W=9, 10, 11, 12 for a BER<10^{-5}.

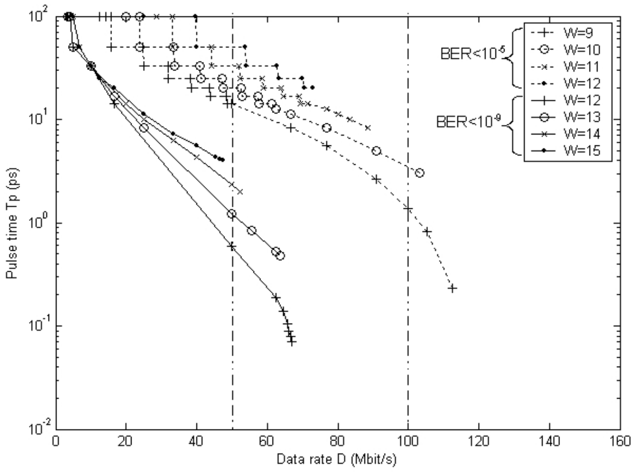

Fig.6: Variations of pulse time and data rate for MWOOC leading to BERs<10^{-9} and BERs<10^{-5}, for several weight values

The curves have been plotted for N=32 active users, a bandwidth B=10*GHz* and for BERs equal to 10^{-5} and 10^{-9}. However, by exploiting the theoretical bound (24), it can be done for any number of users, any electrical bandwidth and any targeted BER.

VI. Conclusion

We have developed in this paper a study of beat noise impact in an incoherent 2-D OCDMA system. Our main contribution is to provide an analytical bound of the performance for incoherent and partially coherent optical regimes. The theoretical error probability expression we have obtained permits evaluating the BER as a function of any system parameter.

Moreover, we have shown that to obtain low BER values, there is a tradeoff between the OCDMA code parameters and the source characteristics. From a parametric study, we have thus assessed the optical source parameters corresponding to a targeted BER and an available data rate.

Considering a simple Conventional Correlation Receiver, we have particularly studied a 2-D OCDMA system using MWOOC with an electrical bandwidth fixed to 10 GHz and N=32 users. However, the theoretical expression can be used for any other code, number of users or bandwidth. Therefore, our work gives the guidelines to follow in order to obtain the system characteristics needed to satisfy a given specification in a beat noise corrupted transmission.

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