(that is, a reduction of about 28% in the convergence time). The gap is even more evident with $\alpha > 0.5$. Similarly, in Fig. 5, the optimum α value is 0.24; $e(k) = 10^{-6}$, for example, requires 5866 transmissions with $\alpha = 0.5$ and "only" 4175 transmissions with $\alpha = 0.24$ (that is, a reduction of about 29% in the convergence time).

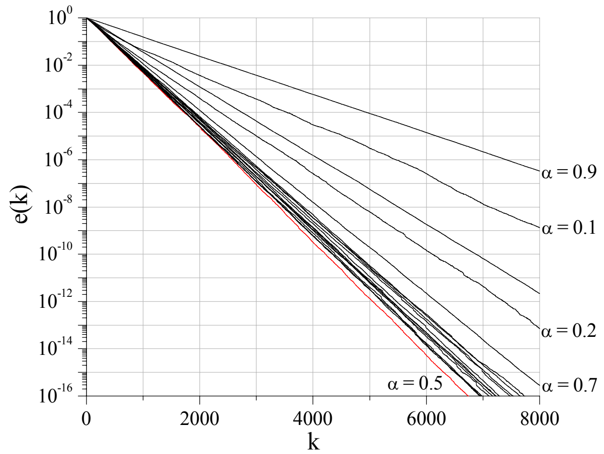


Figure 3. Simulated error for a fully-meshed random geometric graph with $N = 50$, assuming different values of α.

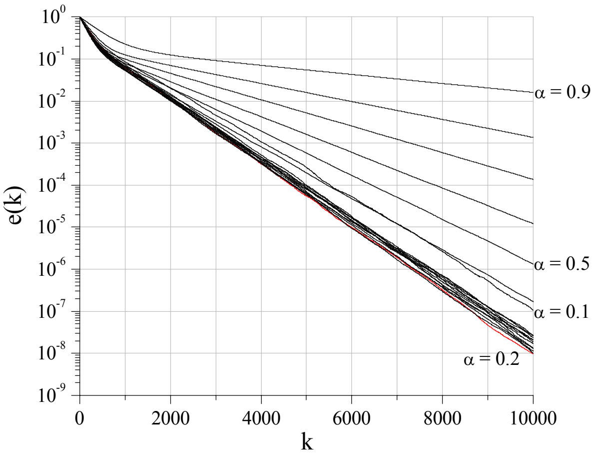


Figure 4. Simulated error for a ring topology with $N = 49$ and $d = 12$, assuming different values of α.

Obviously, to consider a specific simulation gives an idea of the framework, but it is substantially meaningless because of the random nature of the quantities involved. More significantly, from a statistical viewpoint, we can repeat the simulations R times (with R sufficiently high), finding, at any attempt, an optimum value α_{opt}^{m}, with $m = 1, \ldots, R$, and then computing an average optimum share factor as:

$$\left\langle \alpha_{\text{opt}} \right\rangle = \frac{1}{R} \sum_{m=1}^{R} \alpha_{\text{opt}}^{m} . \quad (4)$$

The procedure can be applied for any value of the nodal degree d (or the maximum coverage radius r). Some results are shown in Table I. They have been obtained by averaging the results of 200 simulations for any value of d and r. In the case of the random geometric graph, only values of $r \geq 0.3$ have been considered since, for $r = 0.2$, as an example, the network is no longer connected.

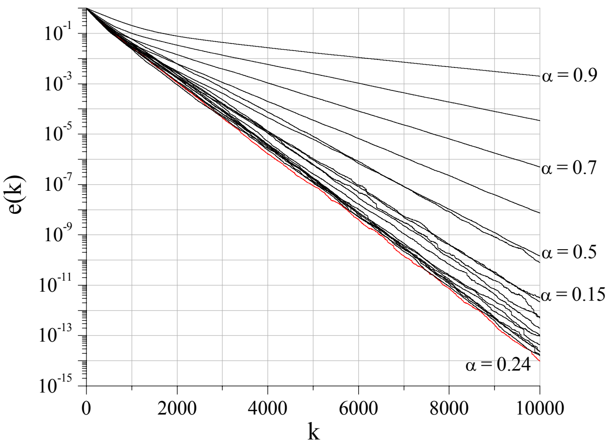


Figure 5. Simulated error for a random geometric graph with $N = 50$ and $r = 0.4$, assuming different values of α.

TABLE I

SIMULATED $\left\langle \alpha_{\text{opt}} \right\rangle$ FOR THE RING NETWORK WITH $N = 49$ AND THE RANDOM GEOMETRIC GRAPH WTH $N = 50$ BY CONSIDERING DIFFERENT CONNECTIVITY LEVELS.

Ring		**Random geometric graph**	
d	$\left\langle \alpha_{\text{opt}} \right\rangle$	r	$\left\langle \alpha_{\text{opt}} \right\rangle$
2	0.29313	0.3	0.2193
6	0.23139	0.4	0.26856
8	0.2202	0.5	0.35428
12	0.22408	0.6	0.43463
16	0.26697	0.7	0.46254
20	0.31667	0.8	0.47164
24	0.37214	0.9	0.46965
28	0.41522	1	0.47672
34	0.45667	1.1	0.47731
48	0.47945	1.2	0.471
		1.3	0.47244
		1.4	0.47289
		1.5	0.48184

On the other hand, (4) provides an average value, while the optimum share for a specific simulation may be more or less different, with a probability that depends on the dispersion around the mean.

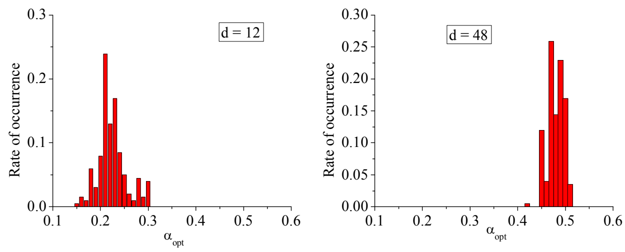
As an example, from Table I we derive that the average optimum share for the case of $d = 12$ is $\left\langle \alpha_{\text{opt}} \right\rangle\big|_{d=12} \approx 0.224$ while for the simulation in Fig. 4 the optimum was $\alpha = 0.2$.

On the other hand, assuming $\left\langle \alpha_{\text{opt}} \right\rangle\big|_{d=12}$ in place of the actual optimum value implies that $e(k) = 10^{-6}$ would have been reached after 7516 transmissions with a limited penalty of less than 2.2% with respect to the optimum value, that is 7356 (see Fig. 4).

In general, the reliability of the average value (4) depends on the dispersion of α_{opt} around the mean. This is shown in Fig. 6, for the ring with $N = 49$, by considering two values of d, and in Fig. 7, for the random geometric graph with $N = 50$ and two values of r.

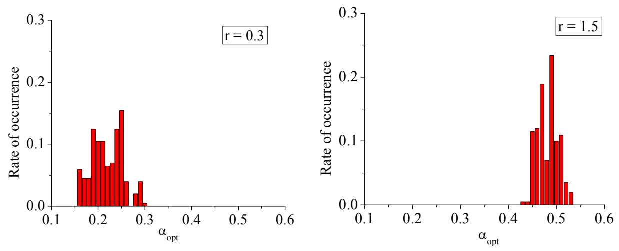
More specifically, the statistical analysis permits us to obtain the normalized standard deviation (nsd), defined as

$nsd = \sqrt{\left\langle \left(\alpha_{\mathrm{opt}} - \left\langle \alpha_{\mathrm{opt}} \right\rangle\right)^2 \right\rangle} / \left\langle \alpha_{\mathrm{opt}} \right\rangle$, that is reported in Table II. We see that, except for the case of very low connectivity, the *nsd* is rather small; so, we can conclude that most networks require an α_{opt} that is not significantly different from the average value.



(a) (b)

Figure 6. Dispersion around the mean of the simulated α_{opt} for the ring network with N = 49 nodes: (a) d = 12; (b) d = 48 (fully-meshed).



(a) (b)

Figure 7. Dispersion around the mean of the simulated α_{opt} for the random geometric graph with N = 50 nodes: (a) r = 0.4; (b) r = 1.5 (fully-meshed).

III. Theoretical Analysis

Besides the simulation work described in the previous section, a theoretical approach has been developed, based on the well known concept of "potential function".

Let us consider a network of N nodes described by a connected graph $G(V, E)$, where V is the vertex set containing the nodes and E is the edge set. Given a distribution of values $\mathbf{v} = [v_1, v_2, ..., v_N]^T$, where v_i is the value of node i, the potential Φ of the graph can be defined, in general, as follows:

$$\Phi = \left\| \mathbf{v} - v_{\mathrm{ave}} \mathbf{1} \right\|^2 = \sum_{i \in V} \left(v_i - v_{\mathrm{ave}} \right)^2 . \quad (5)$$

where v_{ave} is the average value over the whole network. Evidently, Φ is a measurement of the variance of the value distribution. Note that Φ = 0 if and only if $\mathbf{v} = v_{\mathrm{ave}} \mathbf{1}$. In the following we will denote by $\Phi(k)$ the potential function after the k-th clock tick ($\Phi(0)$ is the initial value).

TABLE II
SIMULATED *NSD* FOR THE RING NETWORK WITH N = 49 AND THE RANDOM GEOMETRIC GRAPH WTH N = 50 BY CONSIDERING DIFFERENT CONNECTIVITY LEVELS.

Ring		**Random geometric graph**	
d	*nsd*	*r*	*nsd*
2	0.25551	0.3	0.15276
6	0.19318	0.4	0.07522
8	0.18211	0.5	0.05165
12	0.13477	0.6	0.04257
16	0.06892	0.7	0.05902
20	0.07137	0.8	0.03986
24	0.05294	0.9	0.03449
28	0.04287	1	0.04216
34	0.04949	1.1	0.04735
48	0.03546	1.2	0.04416
		1.3	0.04572
		1.4	0.05329
		1.5	0.04441

Definition (5) could be applied, in principle, directly to the vector of the estimates $\mathbf{x}(k)$, this way obtaining, apart from the normalization by $\mathbf{x}(0)$, the square of the error $e(k)$ as defined in (3). For the push-sum algorithm, however, a slightly more complex expression is more favorable and it is described next.

In order to define the potential function for the push-sum algorithm, let us consider a vector $\mathrm{v}_i(k)$ (that does not contain any measured quantity, but is only introduced for analysis purposes), whose components, $v_{ij}(k)$, are such that:

$$s_i(k) = \sum_{j=1}^{N} v_{ij}(k) x_j(0) . \quad (6)$$

The following condition is satisfied:

$$w_i(k) = \sum_{j=1}^{N} v_{ij}(k) . \quad (7)$$

It is clear that, if $\mathbf{v}_i(k)$ is nearly proportional to the all-one vector, then $x_i(k) = s_i(k)/w_i(k)$ is close to the true average.

The potential function for the push-sum algorithm is defined as follows [5]:

$$\Phi(k) = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[v_{ij}(k) - \frac{w_i(k)}{N} \right]^2 . \quad (8)$$

So, in the limit case of all nodes perfectly aware of the true average, the potential function is null. Based on this evidence, evaluation of the mean potential function, for any k, should permit to estimate the convergence speed of the algorithm.

More precisely, assuming that, at instant k, node l is selected as the transmitter and node m as the receiver, the following difference between the potential functions at time instant k – 1 and k can be easily derived:

$$\delta\Phi = \Phi(k-1) - \Phi(k) = 2\alpha(1-\alpha) \sum_{j=1}^{N} \left[v_{lj}(k-1) - \frac{w_l(k-1)}{N} \right]^2$$
$$-2(1-\alpha) \sum_{j=1}^{N} \left[v_{lj}(k-1) - \frac{w_l(k-1)}{N} \right] \cdot \left[v_{mj}(k-1) - \frac{w_m(k-1)}{N} \right]. \quad (9)$$

For simplifying the notation, in the following we will omit to indicate that the quantities at the right hand side are computed at the time instant $k-1$.

We wish to compute the average of (9) over the possible choices of the transmitting and receiving nodes. Let us consider the case of a ring network, where all nodes are characterized by the same degree d. Taking into account that both the choices of transmitter and receiver are made following a uniform law, the former on the ensemble of the N nodes in the network, and the latter, for each transmitter, on the ensemble of the d nodes it is linked to, also having in mind definition (8), we find:

$$\langle \delta\Phi \rangle = \frac{2\alpha(1-\alpha)}{N}\Phi + \frac{2(1-\alpha)}{dN}\sum_{j=1}^{N}\sum_{l=1}^{N}\left(v_{lj}-\frac{w_l}{N}\right)^2 - \frac{2(1-\alpha)}{dN}\sum_{j=1}^{N}\sum_{l=1}^{N}\sum_{m\in C_l}\left(v_{lj}-\frac{w_l}{N}\right)\left(v_{mj}-\frac{w_m}{N}\right) \qquad (10)$$

where $\Phi = \Phi(k-1)$, and C_l is the subgroup of nodes that includes node l and the nodes it is linked to. It is easy to see that, in the case of a fully-meshed network, when C_l is the entire network and $d = N-1$, the last term at the right side of (10) is null because of the mass conservation properties (1). In the case of a non fully-meshed network, that is $d < N-1$, it represents instead an additional contribution that, in general, could be not simple to determine. However, some elaboration is possible by reminding the definition of Laplacian matrix of a graph [12].

The Laplacian matrix $\mathbf{Q}(G)$ of the graph $G(V,E)$ is an $N\times N$ matrix whose elements are defined as follows:

$$Q_{ij} = \begin{cases} d_i & \text{if } i = j \\ -1 & \text{if } i \neq j \text{ and } (i,j)\in E \\ 0 & \text{otherwise} \end{cases} \qquad (11)$$

where d_i represents the nodal degree of node i. In the case of a ring topology, we have $d_i = d$ for any i. The Laplacian matrix can be also written as $\mathbf{Q} = \boldsymbol{\Delta} - \mathbf{A}$, where $\boldsymbol{\Delta}$ is the diagonal matrix with elements $\Delta_{ii} = d_i$, and $\mathbf{A}$ is the adjacency matrix of the considered graph. The eigenvalues of $\mathbf{Q}$ are called the Laplacian eigenvalues. They are all real and non negative, and satisfy the condition: $0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$. The second smallest eigenvalue, λ_2, is also known as the algebraic connectivity, and it is particularly important. λ_2 is equal to zero only if G is disconnected. Other properties of matrix $\mathbf{Q}$ and its eigenvalues can be found in the literature (see [13], for example).

Let $y_{ij} = v_{ij} - w_i / N$, $i = 1, \ldots, N$, be the components of a vector $\mathbf{y}_j$. Through simple algebra, using (8) and (11), Eq. (10) can be rewritten as follows:

$$\langle \delta\Phi \rangle = -\frac{2(1-\alpha)^2}{N}\Phi + \frac{2(1-\alpha)}{dN}\sum_{j=1}^{N}\mathbf{y}_j^T\mathbf{Q}\mathbf{y}_j \,. \qquad (12)$$

Let us define another vector $\mathbf{z} = (\mathbf{y}_1^T, \mathbf{y}_2^T, \ldots, \mathbf{y}_N^T)^T$ having N^2 components; it is evident that $\mathbf{z}^T\mathbf{z} = \Phi$. Moreover, let us consider a block matrix $\mathbf{L}$, with size $N^2\times N^2$, having N repetitions of $\mathbf{Q}$ along the main diagonal and all the other blocks equal to the null matrix. Also $\mathbf{L}$ can be interpreted as a Laplacian matrix, whose eigenvalues coincide with those of $\mathbf{Q}$, but each appears with multiplicity N.

Using these further definitions, Eq. (12) can be rewritten as:

$$\langle \delta\Phi \rangle = \left[-\frac{2(1-\alpha)^2}{N} + \frac{2(1-\alpha)}{dN}\frac{\mathbf{z}^T\mathbf{L}\mathbf{z}}{\mathbf{z}^T\mathbf{z}}\right]\Phi = \left[-\frac{2(1-\alpha)^2}{N} + \frac{2(1-\alpha)}{dN}\mathrm{RQ}\right]\Phi \qquad (13)$$

having denoted by $\mathrm{RQ} = \mathbf{z}^T\mathbf{L}\mathbf{z} \,/\, \mathbf{z}^T\mathbf{z}$ the so-called Rayleigh quotient. By applying the Courant-Fischer Minimax Theorem [14] it is possible to say that:

$$\lambda_2 \leq \mathrm{RQ} \leq \lambda_N \qquad (14)$$

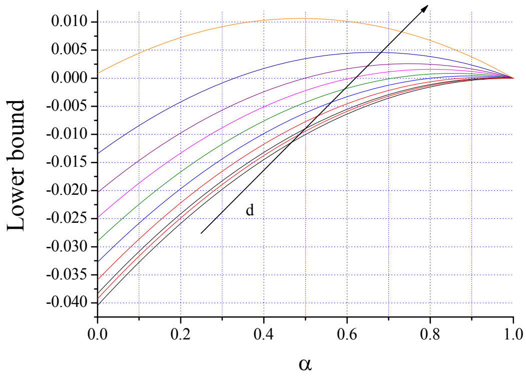
and, consequently:

$$\frac{2(1-\alpha)}{N}\left[-(1-\alpha)+\frac{\lambda_2}{d}\right] \leq \left\langle \frac{\delta\Phi}{\Phi} \right\rangle \leq \frac{2(1-\alpha)}{N}\left[-(1-\alpha)+\frac{\lambda_N}{d}\right]. \qquad (15)$$

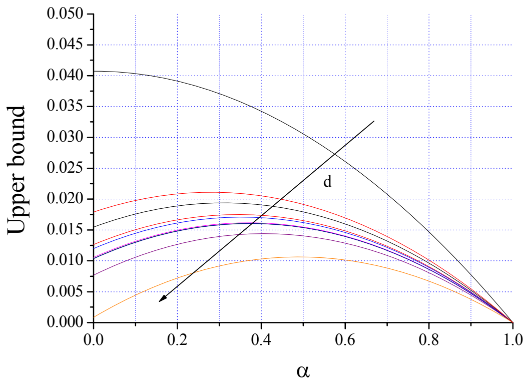
In practice, the above analysis permits us to find a lower bound (*lb*) and an upper bound (*ub*) for the mean variation of the potential function conditioned on a starting value Φ. If the network is fully-meshed, we have *lb* = *ub*, as all eigenvalues, except $\lambda_1 = 0$, are coincident. In this case, the value of $\langle \delta\Phi / \Phi \rangle$ is maximized assuming $\alpha = \alpha_{opt} = (N-2)/(2(N-1)) \approx 0.5$ [11]. To have a maximum $\langle \delta\Phi / \Phi \rangle$ seems a good criterion to make convergence as much faster as possible, and, in fact, simulations confirm that $\alpha \approx 0.5$ is the best choice for fully-meshed network. Additionally, for any $\alpha < 1$, $\langle \delta\Phi / \Phi \rangle > 0$, and this result can be used to demonstrate convergence of the algorithm.

For non fully-meshed networks, instead, *lb* and *ub* show behaviors like those in Fig. 8 for the case of the ring network with $N = 49$. The values of the nodal degree considered are: d = 2, 6, 8, 12, 16, 20, 24, 28, 34, 48. From the figure we see that *lb* becomes lower and lower when reducing d, while *ub* has an opposite behavior. Fig. 8 (a), in particular, shows that convergence of the algorithm cannot be established just on the basis of the *lb* on $\langle \delta\Phi / \Phi \rangle$: for $d < N-1$, in fact, there is a large range of α's where $\langle \delta\Phi / \Phi \rangle < 0$. On the other hand, Fig. 8 (b) shows that reasoning on the *ub* would be too optimistic, since it provides $\langle \delta\Phi / \Phi \rangle > 0$ everywhere, and even greater and greater for decreasing d.

Based on these preliminary results, it is evident that an in depth analysis should weight the contributions of λ_2 and λ_N and, even more important, should take into account the role of the other eigenvalues, λ_i, with $i \in [3, N-1]$.



(a)



(b)

Figure 8. (a) Lower and (b) upper bounds for ⟨δΦ/Φ⟩ in a ring network with $N = 49$ nodes.

With the goal to develop further the theoretical analysis, a first, though coarse, approximation can consist in considering the average of the eigenvalues. An example of the estimate of α_{opt} computed this way is shown in Fig. 9 for the random geometric graph with $N = 50$, where the nodes are located as in Fig. 10; in this case, in Eq. (10) (and in the following ones) $1/d$ has been replaced by $\langle 1/d_i \rangle$. The figure shows how theory can predict the need to reduce the optimum share factor for decreasing $\langle d \rangle$, but the approximations used do not permit a satisfactory agreement between the numerical values. The difference between the theoretical and the simulated results, particularly for small $\langle d \rangle$, can be ascribed to some limits in the analytical approach. In (10), in fact, the subgroup C_l is unconstrained, and the missing links, when the network is non fully-meshed, can be found everywhere in the network. On the contrary, when reduced connectivity is due to the limited maximum coverage radius, the subgroup C_l is constrained and this introduces an effect that is not easy to capture by means of a formula. To confirm such statement, in Section IV, the comparison will be repeated by assuming that missing links due to failures are randomly distributed; and, in that case, we will show that the agreement between the theoretical curve and the simulated results can be better.

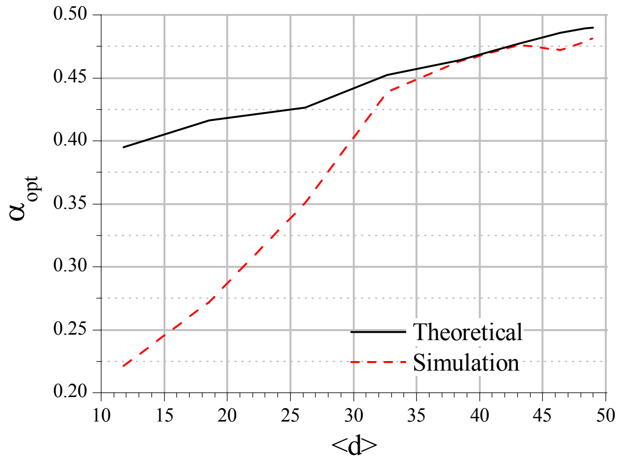


Figure 9. α_{opt} as a function of the average nodal degree for the random geometric graph with $N = 50$ nodes: simulated average vs. analytical estimate.

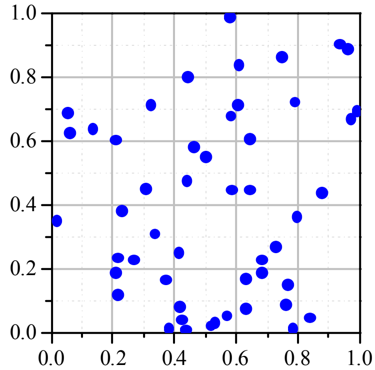


Figure 10. Example of random geometric graph for a network with $N = 50$.

Similar considerations hold for the ring topology. Starting from (13) and maximizing $\langle \delta\Phi / \Phi \rangle$ we find (remind that d is constant for the ring):

$$\alpha_{opt} = 1 - \frac{\mathrm{RQ}}{2d}. \tag{16}$$

In this case, it can be numerically proved that the average of the eigenvalues is almost equal to d. So, the approximate optimum share factor is $\alpha_{opt} \approx 0.5$. Actually, as shown in Fig. 11, the actual optimum value, derived through simulations, can be significantly smaller, particularly for small d. It also shows a minimum, at $d = 8$ in the considered set of values. Even $d = 2$ is acceptable in the figure as, thanks to its structure, the ring network remains connected. Also in this case, the difference between the theory and the simulated results is due to the assumption of constrained links, that is even more evident for the ring topology. By removing this hypothesis, which means to distribute uniformly the missing links, that is a reasonable hypothesis in the case of link failures, the agreement between (16) and the simulated results becomes much better.

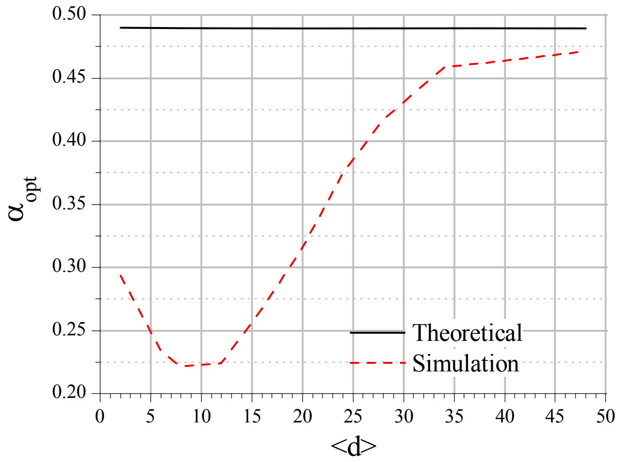


Figure 11. α_{opt} as a function of the nodal degree for the ring topology with N = 49 nodes: simulated average vs. analytical estimate.

IV. The Effect of Failures

When averaging algorithms, like the considered push-sum protocol, are adopted in actual wireless sensor networks, the presence of obstacles or other kinds of radio impairments, as noise and multipath, could prevent some links from being used, due their poor quality in terms of signal-to-noise ratio. In such cases, at least in principle, the network should update the choice of the share factors in order to recover optimal efficiency even in presence of link failures.

However, updating of the share factors could be quite unpractical to implement. Furthermore, the network could be unaware of the number of link failures occurred; so the value of its connectivity degree could not be available for calculating the optimal value of the share factor in the unexpected scenario.

Under these circumstances, it is reasonable to assume that the network will continue to adopt the optimal share factor found for the case without failures. On the other hand, it can be interesting to estimate the performance loss (if any) that results from the adoption of the non optimal value.

Let us consider a fully-meshed random geometric graph; for it, as discussed before, the optimal share, in absence of failures, is about 0.5. Let us suppose that, because of failures, at the beginning of the node interaction, a fraction x of its links, randomly distributed, is missing. While in absence of failures the network connectivity is $N - 1$, in the new situation the average value of d becomes:

$$\langle d \rangle = (N-1)(1-x)\,. \qquad (17)$$

Fig. 12 shows the values of $\langle d \rangle$, calculated through (17), as a function of x, for different choices of N.

If we focus on the curve for N = 50 (i.e., the same considered in the numerical examples of the previous sections), we can notice that, for a fraction of link failures ≤ 0.5, the average nodal degree is still ≥ 25 and, from Fig. 9, we conclude that this implies $\alpha_{opt} \geq 0.35$. With similar arguments, we can derive that, when the fraction of link failures is ≤ 0.3, it is $\alpha_{opt} \geq 0.43$.

Based on this simple analysis, we can guess that, except when the percentage of link failures with respect to the total number of possible links is very high, the presence of obstacles or other impairments in a wireless sensor network based on the push-sum algorithm scarcely affects the choice of the optimal share factor. On the other hand, when link failures occur, the gossip probability becomes smaller than 1, because a node could waste its communication attempt when trying to exploit a failed link.

So, a negative effect on the convergence time is expected, in terms quite similar to those observed in [15] for another version of the gossip algorithm. Our analysis shows that such a penalty cannot be compensated through a different choice of the share factor.

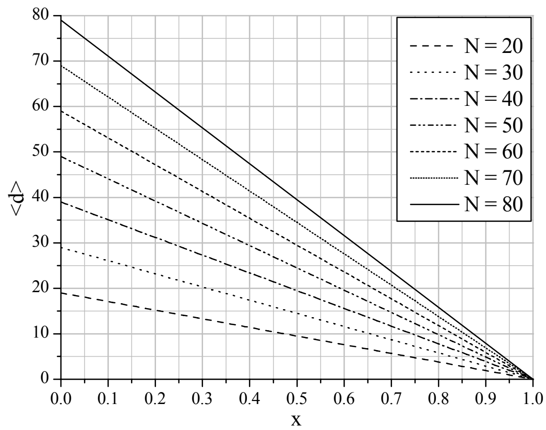


Figure 12. Average value of d for a fully-meshed network in presence of a link failure rate x, for different values of N.

The above considerations do not prevent us to estimate the optimum share factors in presence of link failures for some specific nodes configuration, like that depicted in Fig. 10. This is shown in Fig. 13, where the theoretical curve derived through the approach in Section III is also reported, for the sake of comparison. Fig. 13 is interesting as it demonstrates that the agreement between the analytical approach and the simulated results improves when the missing links are distributed uniformly, which is the hypothesis adopted for simulating link failures. This is because uniform distribution fits well the assumptions implicit in Eq. (10). On the contrary, when reduced connectivity is due to a limited maximum coverage radius, a larger difference is expected, that in fact has been confirmed in Section III.

For the sake of completeness, in Fig. 14 we have reported some values of optimum share factors found for a ring network with N = 49 in presence of link failures. Because of the random distribution of failures, the nodal degree is no longer constant for the ring too, and an average value must be determined. Similarly to Fig. 13, we see that, when missing links are randomly distributed, there is a better correspondence between theoretical expectations and simulations results.

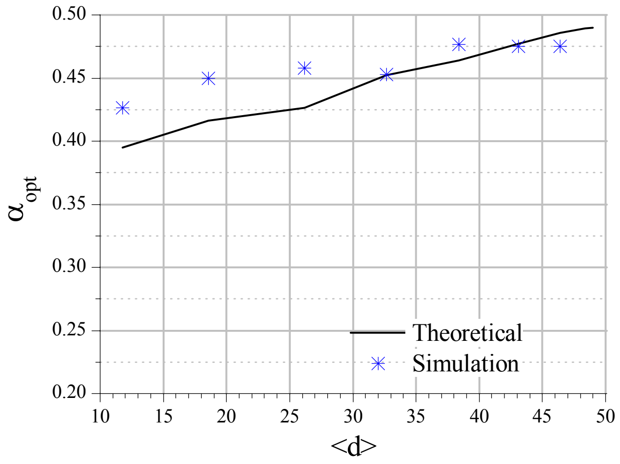


Figure 13. α_{opt} as a function of the average nodal degree for the random geometric graph with $N = 50$ nodes in the presence of link failures: simulated average vs. analytical estimate.

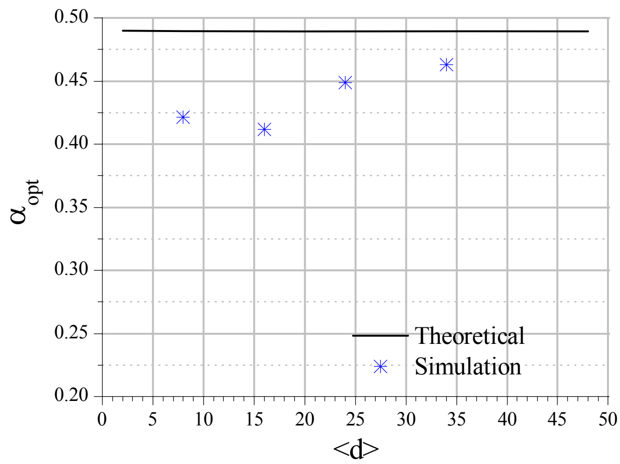


Figure 14. α_{opt} as a function of the average nodal degree for the ring network with $N = 49$ nodes in the presence of link failures: simulated average vs. analytical estimate.

V. CONCLUSIONS

The role of the share factor in the push-sum algorithm has been investigated through simulations and theoretical arguments. Simulations have confirmed that the adoption of an optimal share factor, depending on the network configuration and connectivity level, can improve significantly the convergence speed. Because of the statistical nature of the quantities involved, results have also been given in terms of mean values and standard deviations. We have also shown that the optimal share factor is not significantly influenced by the possible appearance of link failures, at least when the original network has a high connectivity level and the percentage of link failures is not too large.

We have also discussed a first approach, based on the concept of potential function, for a theoretical derivation of the optimal shares. Actually, the possibility to compute analytically the optimum values is very attractive but, at present, only qualitative and approximate information is achievable through simple mathematical arguments. In general, the analytical approach is suitable to describe situations where missing links are uniformly distributed, that is a realistic assumption in the case of random link failures. On the contrary, at least in the current version, it is not able to describe efficiently the constraint induced on links by a limited maximum coverage radius. Improving approximation can consist in overcoming such limitation, as well as in finding a better way for weighting the contributions of different eigenvalues of the Laplacian matrix for the considered graph. This further topic could be the subject of future work.

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